CONSTRUCTION AND RESEARCH OF FULL BALANCE ENERGY OF VARIATIONAL PROBLEM MOTION SURFACE AND GROUNDWATER FLOWS

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Abstract

Based on the laws of conservation of mass and momentum the basic equations of motion with unknown quantities velocity and piezometric pressure are written. These equations are supplemented with boundary and initial conditions describing the motion of compatible flows. Based on the laws of motion continuum, received conditions contact on the common border interaction of surface and groundwater flows. Variational problems formulated compatible flow. Energy norms of basic components of variational problem are analyzed. Correctness of constructing variational problem arising from construction of the energy system of equations that allow to investigate properties of the problem solution, its uniqueness, stability, dependence on initial data and more. Energy equation of motion of surface and groundwater flows are derived and investigated. It is shown that the total energy compatible flow depends on sources that are located inside the domain or on its border.

Keywords: surface flow, groundwater flow, watershed, incompressible fluid, velocity fluid and hydrostatic and piezometric pressure, energy equation, bilinear form, Initial and boundary conditions, interface conditions, coupling flow.

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1. Introduction

An important role in studying the water cycle plays hydrological system. In general, research integrity of the system, taking into account all impacts, are complex and not always feasible problem for the study because only investigated some of the area involved in the water cycle [1–3] Highly likely part of the territory may be a watershed area (**Fig. 1**), which is characterized by similar climatic conditions and is influenced by such factors that affect the water movement.

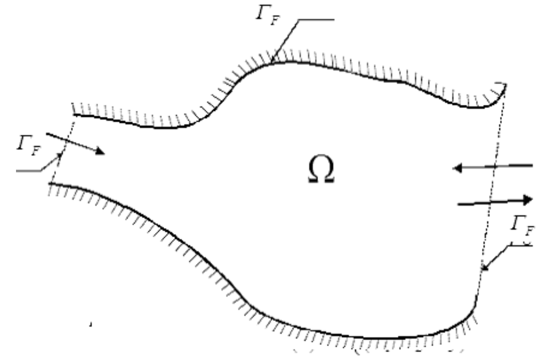

Fig. 1. Two-dimensional projection watershed on the plane X_1OX_2

At the watershed may be an interaction between flow and located above and below water-bearing layers. Models of different dimensions are used in each layer to describe the water movement and their solutions are connected by boundary conditions [4–6].

We select in solid medium (liquid) moving surface layer $F(t)\in R^3$ (**Fig. 2**) of such a structure

$$\Omega_F(t) := \{(x_1, x_2, x_3)\in R^3, \quad \eta(x) < x_3 < \nu(x,t) \quad \forall x = (x_1, x_2)\in \Omega(t)\}. \tag{1}$$

Let's denote projection of its lower

$$\Omega(t):=\left\{(x_1,x_2,x_3)\in R^3 \;\middle|\; x_3=\eta(x),\ \forall x=(x_1,x_2)\in\Omega(t)\right\} \tag{2}$$

and upper

$$\Lambda_F(t):=\left\{(x_1,x_2,x_3)\in R^3 \;\middle|\; x_3=\nu(x,t)\ \ \forall x=(x_1,x_2)\in\Omega(t)\right\} \tag{3}$$

bases on the plane $0x_1x_2$. The rest of the surface layer

$$\Gamma_F(t):=\left\{(x_1,x_2,x_3)\in R^3,\quad \eta(x)<x_3<\nu(x,t)\quad \forall x=(x_1,x_2)\in\Omega(t)\right\} \tag{4}$$

will be called the lateral surface layer $F(t)$.

Similarly denote part of fluid that moves in the soil, so

$$\Omega_P(t):=\left\{(x_1,x_2,x_3)\in R^3,\quad h(x)<x_3<\eta(x),\quad \forall x=(x_1,x_2)\in\Omega(t)\right\} \tag{5}$$

the projection of the lower part will be written as

$$\Lambda_P(t):=\left\{(x_1,x_2,x_3)\in R^3 \;\middle|\; x_3=h(x),\ \forall x=(x_1,x_2)\in\Omega(t)\right\}. \tag{6}$$

Then, a layer of ground water

$$\Gamma_P(t):=\left\{(x_1,x_2,x_3)\in R^3,\quad h(x)<x_3<\eta(x)\quad \forall x\in\Gamma_P(t)\right\}. \tag{7}$$

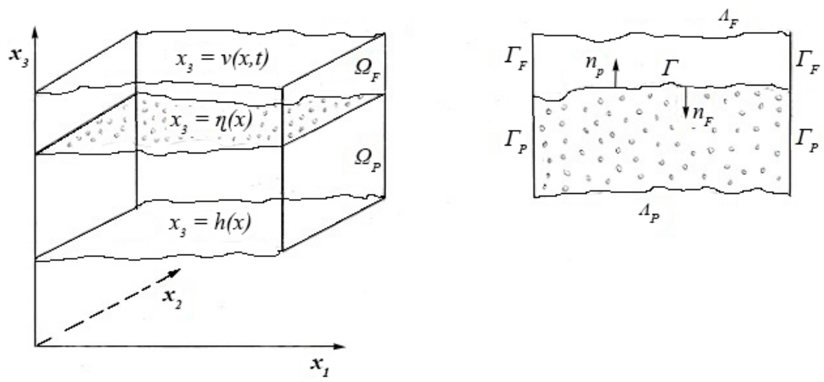

Fig. 2. General view of the model of flows and their cross-section

2. Materials and Methods

2. 1. Initial boundary value problem of interaction of water flows

We formulate initial boundary problem of motion of surface and groundwater flows on the surface watershed considering boundary and initial conditions [7–9].

Find unknown quantities $\{u,p,\varphi\}$ *such that satisfy the following system of equations:*

$$\frac{\partial}{\partial t}(\rho u_i)+\sum_{k=1}^{3}\frac{\partial}{\partial x_k}(\rho\ u_i u_k)-\rho\ f_i-\sum_{k=1}^{3}\frac{\partial\sigma_{ik}}{\partial x_k}=0, \tag{8}$$

$$\sigma_{ij}=-p_F\delta_{ij}+\tau_{ij},$$

$$\tau_{ij}=2\mu\ e_{ij},i,j=1,2,3,$$

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),$$

$$\frac{\partial \rho}{\partial t}+\sum_{k=1}^{3}\frac{\partial(\rho\ u_k)}{\partial x_k}=0,\ \text{in}\quad \Omega_F\times(0,T],\tag{9}$$

$$m\frac{\partial \varphi}{\partial t}=\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(k\frac{\partial \varphi}{\partial x_j}\right)+\varepsilon\ \text{ in }\ \Omega_P\times(0;T],\tag{10}$$

where $\{u_i(x,t)\}^3_{i=1}$ and $p_F=p_F(x,t)$ – sought velocity vector of fluid and hydrostatic pressure, respectively; $F=\{g\ f_i(x)\}_{i=1}^{3}$ – mass forces; ρ=ρ(x,t)>0 – density of the mass water flow; μ=μ(x)>0 – viscosity coefficient; $\{e_{ij}\}_{i,j=1}^{3}$, $\{\sigma_{ij}\}_{i,j=1}^{3}$ – tensors of velocities of deformation and stress of the liquid at the point x in time t; δ_{ij} – Kronecker symbol; $k=k(x,t)$ – filtration coefficient; $m=m(x,t)$ – coefficient of specific water loss; $\varepsilon=\varepsilon(x,t)$ – known function of sources of water influx;

$$\varphi=x_3+\frac{p_P}{\rho g}\tag{11}$$

piezometric pressure;

$$q=-k\nabla\varphi\tag{12}$$

flow (flow separation); $\upsilon=\upsilon(x,t)$ – velocity vector of fluid in the ground; $\upsilon=\frac{q}{\omega}$, ω – volume porosity; $\overrightarrow{n_F}=-\overrightarrow{n_P}$ – vectors normal to the boundary area Ω_F and Ω_P in accordance;

$$\overline{\Omega}=\overline{\Omega_F}\cup\overline{\Omega_P},\Omega_F\cap\Omega_P=\{\varnothing\},\overline{\Omega_F}\cap\overline{\Omega_P}=\Gamma,$$

$$\partial\Omega_F=\Gamma_F\cup\Lambda_F\cup\Gamma;\partial\Omega_P=\Gamma_P\cup\Lambda_P\cup\Gamma.$$

Boundary conditions [10, 11]:

$$\vec{u}_i=0\ \text{ on }\ \Gamma_F,\ i=1,2,3,\tag{13}$$

$$\sigma_{n\tau}=\overline{\sigma},\ \text{on}\ \Lambda_F,\tag{14}$$

$$u_3+R=\frac{\partial \nu}{\partial t}+u_1^0\frac{\partial \nu}{\partial x_1}+u_2^0\frac{\partial \nu}{\partial x_2}\ \text{ in }\ \Omega_F\times(0,T],\tag{15}$$

where R – velocity of falling rain drops, u_1^0, u_2^0 – horizontal components of velocity on the free surface $\nu(x.t)$ (Λ_F);

$$\upsilon_\bullet\ n_p=\overline{\upsilon}\quad \text{on}\quad \Gamma_P;\tag{16}$$

$$\upsilon_1=\upsilon_2=0\quad \text{on}\quad \Lambda_P,\tag{17}$$

$$\upsilon_3=-I\ \text{ on }\ \Lambda_P,\tag{18}$$

where I – known function that describes the velocity of fluid flow through the surface Λ_P.

Initial conditions:

$$\begin{aligned}&u|_{t=0}=u_0,\\&p|_{t=0}=p_0,\ \text{ in }\Omega.\\&\varphi|_{t=0}=\varphi_0,\end{aligned}\tag{19}$$

Contact flow conditions on a common boundary Γ [4–6, 8]:

$$\sigma_{nn}(u, p_F) = p_p,$$

$$\sigma_{\tau n} = 0, \qquad (20)$$

$$u_n = -\upsilon_n.$$

2. 2. Variational formulation of the problem of interaction of water flows

We introduce the following bilinear forms:

$$M_v(r; w, q) = \int_v \sum_{i=1}^{3} r w_i q_i ds, \quad N_v(w; u, q) = \int_v \sum_{k=1}^{3}\sum_{i=1}^{3} \rho w_k \frac{\partial u_i}{\partial x_k} q_i ds,$$

$$C_v(w, q) = \int_v 2\mu \; e(w) : e(q) ds,$$

$$A_v(w, q) = -\int_v w \operatorname{div} q ds, \quad Y_v(w, q) = -\int_v w q_n d\gamma, \quad B_v(p, w) = -\int_v \sum_{i=1}^{3} p.\nabla w ds.$$

Introduce spaces:

$$H_F := \left\{\xi \in (H^1(\Omega_F))^3 \mid \xi = 0 \;\; \text{on} \;\; \Gamma\right\},$$

$$H_p := \left\{\psi \in H^1(\Omega_p) \mid \psi = 0 \;\; \text{on} \;\; \Gamma\right\}, \quad W := H_F \times H_P, \; \Im_j : W \to R, j = \overline{1,3},$$

$$\left\langle \Im_1, \xi \right\rangle = \sum_{i=1}^{3} \int_{\Omega_F} \rho f_i \xi_i ds + \int_{\Lambda_F} (\xi_n p_a + \xi_\tau \cdot \widehat{\sigma}) d\gamma,$$

$$\left\langle \Im_2, \theta \right\rangle = -\int_{\partial\Lambda_F} u_n^0 \theta d\gamma, \; \left\langle \Im_3, \psi \right\rangle = \int_{\Omega_P} \frac{\varepsilon(x,t)\rho g \psi}{\omega} dp - \int_{\partial\Lambda_P} \overline{\upsilon}\, \psi \rho g d\gamma.$$

Let's denote

$$\tilde{\psi} = \psi\rho g, \quad \tilde{m} = \frac{m}{\omega},$$

Then, let's write the following variational problem [1–2, 10, 11]:

$$\text{Find}\left\{u, p, \varphi\right\} \in V \times Q \times W,$$
$$M_{\Omega_F}(\rho; u', \xi) + N_{\Omega_F}(u; u, \xi) + A_{\Omega_F}(p, \xi) + C_{\Omega_F}(u, \xi) +$$
$$+ Y_\Gamma(u, \xi) = \left\langle \Im_1, \xi \right\rangle, \forall \xi \in V, \qquad (21)$$

$$B_{\Omega_F}(u, \theta) + Y_\Gamma(\theta, u) = \left\langle \Im_2, \theta \right\rangle, \forall \theta \in Q, \qquad (22)$$

$$M_{\Omega_P}(\tilde{m}; \varphi', \tilde{\psi}) + A_{\Omega_P}(\tilde{\psi}, \upsilon) + Y_\Gamma(\tilde{\psi}, \upsilon) = \left\langle \Im_3, \tilde{\psi} \right\rangle, \forall \psi \in W \qquad (23)$$

with initial conditions

$$M_{\Omega_F}(u'(0) - u_0, \xi) = 0, \qquad (24)$$

$$B_{\Omega_F}(p(0) - p_0, \theta) = 0; \qquad (25)$$

$$M_{\Omega_P}(\varphi'(0) - \varphi_0, \tilde{\psi}) = 0. \qquad (26)$$

Let's calculate, considering initial conditions (24)–(26) and boundary conditions (13)–(18), values of variables u and p with relations (21) and (22). Then on the basis of coupling flow conditions (interface conditions) (20) and boundary condition (11) the value of the variable φ is calculated from (23).

2. 3. The properties of the components and norms of variational problem interaction water flows.

It should be noted that trilinear form

$$N_v(w;u,q)=\int_v\sum_{k=1}^{3}\sum_{i=1}^{3}\rho w_k\frac{\partial u_i}{\partial x_k}q_i ds, \tag{27}$$

is continuous and bilinear form

$$C_v(w,q)=\int_v 2\mu e(w):e(q)\,ds \tag{28}$$

continuous and symmetrical.

It is a scalar product in the space H_F and creates a norm

$$\|w\|_{H_F}=\sqrt{C_v(w,w)},\forall w\in H_F.$$

Then, let's write the scalar function φ bilinear forms

$$D_v(\varphi,\psi)=\int_v k(x,t)\nabla\varphi\cdot\nabla\psi\ dp. \tag{29}$$

which is continuous and integral in the space of admissible functions H_P. It is also symmetrical and forms a semi-norm

$$|\varphi|_{H_P}=\sqrt{D_v(\varphi,\varphi)},\forall\varphi\in H^1(\Omega_P). \tag{30}$$

Let's consider the properties of bilinear forms

$$A_v(w,q)=\int_v \mathrm{div}w\,\mathrm{div}q\,ds. \tag{31}$$

In space H_F, it is continuous, integral and symmetrical, and also forms the norm

$$|||q|||_{H_F}=\sqrt{A_v(q,q)},\forall q\in H_F. \tag{32}$$

3. Results of research

3. 1. Equation balance energy of coupling water flow

Let's write variational equations for momentum

$$M_{\Omega_F}(\rho;u',u)+N_{\Omega_F}(u;u,u)+C_{\Omega_F}(\mu;u,u)=$$
$$=\langle\varphi_1,u\rangle-Y_{\Omega_F}(p,u)-A_{\Omega_F}(p,u). \tag{33}$$

Let's write left side of the equation:

$$\int_{\Omega_F}\sum_{i=1}^{3}\rho_i u_i' u_i ds+\int_{\Omega_F}\sum_{k=1}^{3}\sum_{i=1}^{3}\rho u_k\frac{\partial u_i}{\partial x_k}u_i ds-\int_{\Omega_F}\sum_{k=1}^{3}\frac{\partial\sigma_{ik}}{\partial x_k}u_k ds=\int_{\Omega_F}\sum_{i=1}^{3}\rho_i u_i' u_i ds+$$
$$+\int_{\Omega_F}\sum_{k=1}^{3}\sum_{i=1}^{3}\rho u_k\frac{\partial u_i}{\partial x_k}u_i ds+\int_{\Omega_F}\sum_{k=1}^{3}\sigma_{ik}\frac{\partial u_i}{\partial x_k}ds-\int_{\partial\Omega_F}u_i\sum_{k=1}^{3}\sigma_{ik}n_{F_k}d\gamma. \tag{34}$$

Given that

$$\sigma_{ik} = -p\delta_{ik} + 2\mu e_{ik}$$

rewrite (34) in the following form

$$\int_{\Omega_F} \sum_{i=1}^{3} \rho_i u_i' u_i ds + \int_{\Omega_F} \sum_{k=1}^{3} \sum_{i=1}^{3} \rho u_k \frac{\partial u_i}{\partial x_k} u_i ds - \int_{\partial\Omega_F} p\ u_n d\gamma +$$
$$+ \int_{\Omega_F} u\ \nabla p ds + \int_{\Omega_F} 2\mu\, e(u) : e(u)\, ds\ - \int_{\partial\Omega_F} u_i \sum_{k=1}^{3} \sigma_{ik} n_{F_k} d\gamma. \qquad (35)$$

Let's write left side of variational equations for the law of conservation of mass flow

$$\int_{\Gamma} u.n_F p d\gamma - \int_{\Omega_F} u.\nabla p ds = 0. \qquad (36)$$

Substituting (23) in place of ψ the function φ will be

$$\int_{\Omega_p} m \frac{\partial\varphi}{\partial t} \varphi \frac{\rho g}{\omega} dp - \int_{\partial\Omega_p} k \frac{\partial\varphi}{\partial n_p} \varphi d\gamma +$$
$$+ \int_{\Omega_p} \sum_{j=1}^{3} m \frac{\partial\varphi}{\partial x_j} \frac{\partial\varphi}{\partial x_j} dp - \int_{\Omega_p} \varepsilon\varphi dp = 0. \qquad (37)$$

Let's multiply (37) on the expression $\frac{\rho g}{\omega}$, then

$$\frac{1}{2} \int_{\Omega_p} m \frac{\partial\varphi^2}{\partial t} \frac{\rho g}{\omega} dp - \int_{\partial\Omega_p} \sum_{j=1}^{3} \frac{k(x,t)}{\omega} \frac{\partial\varphi}{\partial \overline{n}} \varphi\ \rho g d\gamma +$$
$$+ \int_{\Omega_p} \sum_{j=1}^{3} \frac{k(x,t)}{\omega} \frac{\partial\varphi}{\partial x_j} \frac{\partial\varphi}{\partial x_j}\ \rho g\ dp - \int_{\Omega_p} \frac{\rho g}{\omega} \varepsilon\varphi dp = 0. \qquad (38)$$

Let's estimate the term in (38) on the boundary Ω_p

$$-\int_{\partial\Omega_p} \frac{k}{\omega} \frac{\partial\varphi}{\partial n_p} \varphi\rho g d\gamma = -\int_{\Gamma_p} \frac{k}{\omega} \frac{\partial\varphi}{\partial n_p} \varphi\rho g d\gamma - \int_{\Gamma} \frac{k}{\omega} \frac{\partial\varphi}{\partial n_p} \varphi\rho g d\gamma - \int_{\Lambda_p} \frac{k}{\omega} \frac{\partial\varphi}{\partial n_p} \varphi\rho g d\gamma =$$
$$= -\int_{\Gamma} k\varphi\rho g \frac{\nabla\varphi n_p}{\omega} d\gamma - \int_{\Gamma_p} k \frac{\nabla\varphi n_p}{\omega} \varphi\rho g d\gamma =$$
$$= \int_{\Gamma} p_p \upsilon_{n_p} d\gamma - \int_{\Gamma_p} \overline{\upsilon}\ \varphi\rho g d\gamma. \qquad (39)$$

Simplifying a term on the border Ω_F in the form (35), we obtain

$$-\int_{\partial\Omega_F} u_i \sum_{k=1}^{3} \sigma_{ik} n_{F_k} d\gamma = -\int_{\Lambda_F} (u_n\sigma_{nn} + u_\tau\sigma_{n\tau}) d\gamma -$$
$$-\int_{\Gamma_F} (u_n\sigma_{nn} + u_\tau\sigma_{n\tau}) d\gamma - \int_{\Gamma} (u_n\sigma_{nn} + u_\tau\sigma_{n\tau}) d\gamma. \qquad (40)$$

Adding expressions (35), (39), (40), after simple transformations, we have

$$\int_{\Omega_F}\sum_{i=1}^{3}\rho_i u_i' u_i dx+\int_{\Omega_F}\sum_{k=1}^{3}\sum_{i=1}^{3}\rho u_k\frac{\partial u_i}{\partial x_k}u_i dx-$$

$$-\int_{\Lambda_F} p\ u_n d\gamma-\int_{\Gamma_F} p\ u_n d\gamma-\int_{\Gamma} p\ u_n d\gamma+\int_{\Omega_F} u\ \nabla p\ ds+\int_{\Omega_F} 2\mu\, e(u):e(u)\,dx-$$

$$-\int_{\Lambda_F}(u_n p_a+u_\tau\sigma_{n\tau})d\gamma-\int_{\Gamma}(u_n\sigma_{nn}+u_\tau\sigma_{n\tau})d\gamma+\int_{\Gamma}u.n_F p d\gamma-\int_{\Omega_F}u.\nabla p ds+$$

$$+\frac{1}{2}\int_{\Omega_p}\frac{m\rho g}{\omega}\frac{\partial\varphi^2}{\partial t}dx+\int_{\Omega_p}\sum_{j=1}^{3}\frac{k(x,t)}{\omega}\left(\frac{\partial\varphi}{\partial x_j}\right)^2\rho g\ dp-\int_{\Omega_p}\frac{\rho g}{\omega}\varepsilon\varphi dx-$$

$$-\int_{\Gamma}p_p\upsilon_{n_p}d\gamma+\int_{\Gamma_p}\bar{\upsilon}\ \varphi\rho g d\gamma=0. \qquad (41)$$

Rewriting the previous expression (41) in a more convenient form, including property incompressible environment and boundary conditions (13)–(18), we obtain

$$\int_{\Omega_F}\sum_{i=1}^{3}\rho_i u_i' u_i dx+\int_{\Omega_F}\sum_{k=1}^{3}\sum_{i=1}^{3}\rho u_k\frac{\partial u_i}{\partial x_k}u_i dx+\int_{\Omega_F}2\mu\, e(u):e(u)\,dx-$$

$$-\int_{\Lambda_F}p\ u^0{}_n d\gamma-\int_{\Lambda_F}(u_n p_a+u_\tau\bar{\sigma})d\gamma-$$

$$-\int_{\Gamma}(u_n\sigma_{nn}+u_\tau\sigma_{n\tau})d\gamma+\frac{1}{2}\int_{\Omega_p}\frac{m\rho g}{\omega}\frac{\partial\varphi^2}{\partial t}dx+\int_{\Omega_p}\sum_{j=1}^{3}\frac{k(x,t)}{\omega}\left(\frac{\partial\varphi}{\partial x_j}\right)^2\rho g\ dp-$$

$$-\int_{\Omega_p}\frac{\rho g}{\omega}\varepsilon\varphi dx-\int_{\Gamma}p_p\upsilon_{n_p}d\gamma+\int_{\Gamma_p}\bar{\upsilon}\ \varphi\rho g d\gamma=0. \qquad (42)$$

Let's analyze the terms on joint border Γ

$$\int_{\Gamma}(u_n p_F+u_\tau\sigma_{n\tau}(u)-p_p\upsilon_{n_p})d\gamma.$$

Given the terms the coupling (20), integral to the common border Γ is zero.

From the expression (36) given kinematic condition (15) for equation of continuity will be

$$\int_{\partial\Omega_F}u.n_F p d\gamma=\int_{\Lambda_F}u_n^0\ p d\gamma. \qquad (43)$$

Thus, the energy balance equation of compatible motion of surface and groundwater flow is written:

$$\int_{\Omega_F}\sum_{i=1}^{3}\rho_i u_i' u_i dx+\int_{\Omega_F}\sum_{k=1}^{3}\sum_{i=1}^{3}\rho u_k\frac{\partial u_i}{\partial x_k}u_i dx+\int_{\Omega_F}2\mu\, e(u):e(u)dx-$$

$$+\frac{1}{2}\int_{\Omega_p}\frac{m\rho g}{\omega}\frac{\partial\varphi^2}{\partial t}dx+\int_{\Omega_p}\sum_{j=1}^{3}\frac{k(x,t)}{\omega}\left(\frac{\partial\varphi}{\partial x_j}\right)^2\rho g\ dp=$$

$$\sum_{i=1}^{3}\int_{\Omega_F}\rho f_i u_i ds+\int_{\Lambda_F}p\ u^0{}_n d\gamma-$$

$$-\int_{\Gamma_p}\bar{\upsilon}\ \varphi\rho g d\gamma+\int_{\Lambda_F}(u_n p_a+u_\tau\bar{\sigma})d\gamma+\int_{\Omega_p}\frac{\rho g}{\omega}\varepsilon\varphi dx. \qquad (44)$$

Rewriting (44) through a total derivative, we have

$$\frac{1}{2}\rho\int_{\Omega_F}\frac{d}{dt}(u^2)\ ds+\int_{\Omega_F}2\mu\, e(u):e(u)dx-$$

$$+\frac{1}{2}\int_{\Omega_p}\frac{m\rho g}{\omega}\frac{\partial\varphi^2}{\partial t}dx+\int_{\Omega_p}\sum_{j=1}^{3}\frac{k(x,t)}{\omega}\left(\frac{\partial\varphi}{\partial x_j}\right)^2\rho g\ dp=$$

$$\sum_{i=1}^{3}\int_{\Omega_F}\rho f_i u_i ds+\int_{\Lambda_F}p\ u^0{}_n d\gamma-$$

$$-\int_{\Gamma_p}\overline{\upsilon}\ \varphi\rho g d\gamma+\int_{\Lambda_F}(u_n p_a+u_\tau\overline{\sigma})d\gamma+\int_{\Omega_p}\frac{\rho g}{\omega}\varepsilon\varphi dx. \qquad (45)$$

As we see from (45), the total energy flow depends on the energy sources that are located within the region or within its boundaries.

4. Conclusions

On the basis of conservation laws basic equations and boundary and initial conditions are derived describing the compatible motion flow of surface and ground water with unknown values of velocity and piezometric pressure. Variational problems of compatible flow are formulated and the contact conditions on the common border are obtained based on the laws of motion continuum. Energy standards of basic components of variational problem are analyzed. Full energy equation of energy balance for coupling motion of surface and groundwater flows are constructed and studied that makes it possible to investigate the properties of solutions of the problem, such as stability, regularity, existence, convergence and so on.

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