DETERMINATION OF THE HEAT DISTRIBUTION IN THE RAW COTTON PACKED IN THE COIL

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Abstract

As a result of experimental studies, a special mathematical model of raw cotton is developed. The effect of density change on the thermal conductivity coefficient is determined. A nonlinear differential equation of heat propagation in coils is obtained. The dependence of the density of raw cotton on the coil height is determined experimentally. The heat flux is intense propagating from denser layers of raw cotton to less dense ones. In a saturated form, the effect of density changes on the propagation of heat is less than in the coils. Pocket spontaneous heating occurs locally with sharp boundaries.

An expression is found, which is the general solution of the mathematical model of heat propagation in raw cotton in coils, on the basis of which a number of physical real models can be constructed.

The model allows to preliminarily give an estimation of the likely picture of the temperature field in the given microvolumes of raw cotton.

Keywords: raw cotton, spontaneous heating, oil content, thermal conductivity coefficient, mathematical model of heat propagation, temperature field.

1. Introduction

Harvesting of raw cotton, depending on weather conditions, lasts several weeks. About 20 % of harvested raw cotton is processed by cotton ginning plants during the harvesting season, however, the raw cotton still have to be stored for long storage, for processing it in the next months [1].

When storing raw cotton with moisture above 10 % of I and II grades and 13 % for III and IV, its spontaneous heating occurs.

As a result of spontaneous heating, there is a decrease in the absolutely dry mass of warming cotton. The quality of the fiber deteriorates, the breaking load decreases, resulting in the fiber grade being converted to low grades. The oil content of seeds is reduced by 50–55 % due to burnt fractions.

By creating the most optimal conditions during the storage of raw cotton, it is possible to prevent the spontaneous heating and to achieve a certain effect in improving the quality of cotton components.

Therefore, preventing of the spontaneous heating of raw cotton in a coil is an urgent task.

The problem of qualitative storage of raw cotton is given serious attention on the part of specialists in the primary processing of cotton.

The physical essence of the spontaneous heating of raw cotton is so complex that the authors of the works on primary cotton processing [2–4] only state the occurrence of spontaneous heating at high humidity and mainly its consequences are considered.

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So [5] describes the biological process occurring inside the seed of raw cotton.

In [6], studies are carried out in laboratory conditions on the spontaneous heating of cotton of I–IV grades at a bulk density of 2200–2500 N/m^3 from the initial humidity, which changed from equilibrium to 50.5 %. As a result of the experiments, the temperature rise of raw cotton is depend on its moisture content with the simultaneous loss of an absolutely dry mass of warming cotton.

In [7], the author explains the nature of spontaneous heating of the vital functions of seeds.

According to [7, 8], spontaneous heating of cotton comes at an extreme humidity, manifested through the intermediary-developing microorganisms.

2. The aim and objectives of research

The aim of research is to obtain a mathematical model of heat propagation in raw cotton when stored in coils.

To achieve the aim, the following tasks are set:

– to study the possibility of propagation of the activity front with a constant speed;

– to develop a special mathematical model of raw cotton on the basis of experimental studies;

– to study the effect of the density of raw cotton on the thermal conductivity coefficient;

– to obtain the heat transfer equation for the case of the bulk state of raw cotton.

3. Results of studies to determine the heat propagation in the raw cotton

Let's take the heat released by the active biological medium as the main indicator of the spontaneous heating of cotton in the coil. In this case, the equation of the mathematical model is written by a nonlinear kinetic equation:

$$\rho c = \frac{\partial U}{\partial t} = \Delta(\lambda U) + f(U) - v_0 \frac{\partial U}{\partial x} \cdot \rho c, \tag{1}$$

where v_0 – the rate of propagation of the activity front of the biological environment of raw cotton, which characterizes the process of self-organization (the rate of spontaneous heating in the mass of raw cotton); $f(U)$ – the given function; Δ – the Laplace operator.

Let's consider the special case of the form of the function $f(u)$

$$f(U) = -h(U - U_0),$$

where U_0 – the initial temperature of the medium, h – linear approximation coefficient.

In this case, equation (1) is not an equation for self-organization, it describes only heat losses. Therefore, it is necessary to include a nonlinear term in the equation that is balanced with the dissipative term, as a result of which the development of the process is ensured.

Therefore, in (1) let's set

$$f(U) = -h(U - U_0) + h_1(U - U_0)^2,$$

then (1) can be rewritten as:

$$c\rho = \frac{\partial U}{\partial t} = \Delta(\lambda U) - h(U - U_0) + h_1(U - U_0)^2 - v_0 \frac{\partial U}{\partial x} \cdot \rho c, \tag{2}$$

where h_1 – second approximation coefficient.

Let's pose the question of the propagation possibility of the activity front with a constant velocity V_0.

The temperature of the medium is determined by formula

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$$U - U_0 = \frac{3h}{h_i} \cdot \frac{1}{ch^2\left[\sqrt{\frac{h}{2\lambda}}(x - V_0 t)\right]}, \quad (3)$$

where chz – the hyperbolic cosine of the function, t – time, s.

The graph of function (3) is shown schematically in **Fig. 1**.

$$U_{max} - U_0 = \frac{3h}{h_1},$$

$$2h_1 \cdot h = \lambda \cdot V_0.$$

When these conditions are fulfilled, a front of activity is formed in the medium, moving (propagating) at a constant rate V_0.

The temperature accumulated in the source, as a result of kinetic changes in biological changes, initially has a local character, and then, until a certain period of time, develops and spreads around the source.

Moisture in raw cotton can be in the form of a free vapor between the fibers, as a bond between the surfaces of the fibers and the form of moisture inside the fibers and seeds (**Fig. 1**) [9].
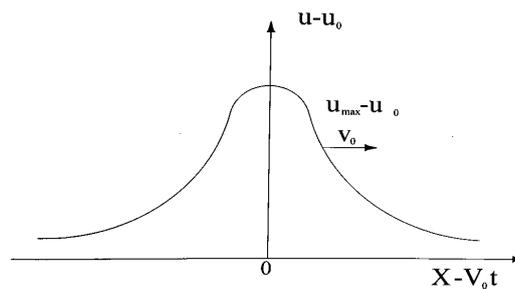

Fig. 1. Graphical representation of the function $U - U_0$ variation from the value $X-V_0 t$

Therefore, large changes in temperature are necessarily accompanied by large qualitative changes in the physical parameters of raw cotton. Physical processes in the mass of raw cotton are non-uniform, non-stationary and non-linear [1]. Therefore, there is a need to develop a special mathematical model of raw cotton on the basis of experimental studies. Thus, the thermophysical properties of raw cotton can be written as an empirical formula.

$$\lambda = \left[4 \cdot 10^{-2} + 4 \cdot 10^{-4} \cdot U + 1,1 \cdot 10^{-4} \cdot \rho + 10^{-2} \cdot 0,11\{7\,\% - w\}\right], \quad (4)$$

where λ – heat conductivity of raw cotton, kcal/ m·h; W – moisture content, %; U – temperature, °C; ρ – volumetric mass, kg/m^3.

The bulk density of raw cotton in a coil is described by equality

$$\rho = \rho_0\left(1 + \frac{\rho'}{\rho_0}\right),$$

where ρ – the current bulk density, N/m^3; ρ_0 – the average density value; ρ' – deviation from the average value.

To describe the propagation of a given initial heat, let's use the known differential equation (12)

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$$\rho \cdot c \cdot U_t = \Delta\,(\lambda U), \qquad (5)$$

$$U/_{t=o} = \varphi(x,y,z),$$

where c – heat capacity, J/K; Δ – Laplace operator; $\varphi(x,y,z)$ – initial heat distribution.

Experimental studies have shown that in the initial area of spontaneous heating, raw cotton has a temperature of not more than $U/_{t=o}$=80 ºC, and in a sufficiently remote region from the heat source U=20 ºC. The relative humidity of raw cotton W=14 %.

Following the known methods of investigating equation (5), let's introduce dimensionless functions and arguments:

$$X' = \frac{X}{M}\,;\ Y' = \frac{Y}{M}\,;\ Z' = \frac{Z}{M}\,;\ U' = \frac{U}{U_{orta}}\,. \qquad (6)$$

Then

$$U_{max} = 80\ ^{\mathrm{o}}C,\ U_{min} = 20\ ^{\mathrm{o}}C,$$

$$U = U_{min}\,(1+3V), \qquad (7)$$

where V – a dimensionless function.

Let's consider the raw cotton within the temperature range U from U_{min} to U_{max}. The relationship between them will be

$$U = U_{av} \cdot \left(1 + \frac{U'}{U_{av}}\right);\ U_{av} = U_{max} - U_{min}, \qquad (8)$$

$$U_{cp} = U_{min} + \frac{U_{max} - U_{min}}{2}.$$

For example, if U_{max}=80 ºC; U_{min}=20 ºC, then U_{av}=50 ºC,

$$U = 50\ ^{\mathrm{o}}C\left(1 + \frac{U'}{50\ ^{\mathrm{o}}C}\right),$$

because

$$U' \le 30\ ^{\mathrm{o}}C,\ \ \frac{U'}{50^{\mathrm{o}}} \le 0{,}75,$$

If introduce the dimensionless function U so that U=80 ºC, then the dimensionless function V=1, at U=20 ºC, V=0.

In this case, the dimensionless function U takes the form

$$U = 20^{\mathrm{o}}(1+3V). \qquad (9)$$

Transforming expression (5) let's obtain

$$\lambda = 1{,}7 \cdot 10^{-2}\left(1 + V + 0{,}55\frac{\rho'}{\rho_0}\right). \qquad (10)$$

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The effect of density variation on the thermal conductivity is much less than the effect of temperature and it can be attributed to the third order of smallness. Taking the second approximation, let's obtain

$$\lambda = 1{,}7 \cdot 10^{-2} \ (1+V). \tag{11}$$

From this analysis let's obtain a nonlinear differential equation of heat propagation in coils.

$$\left(1 + \frac{\rho^{/}}{\rho_0}\right)\rho_0 \cdot cV_t = 1{,}7 \cdot 10^{-2} \Delta \left\{V + V^2 + 0{,}55V\frac{\rho^{/}}{\rho}\right\}. \tag{12}$$

Thus, for $t^{/}$ in dimensionless variables:

$$t^{/} = t \cdot \frac{1{,}7 \cdot 10^{-2}}{\rho_0 \cdot c}. \tag{13}$$

Then expression (12) takes the form:

$$\left(1 + \frac{\rho^{/}}{\rho_0}\right)V_t = \Delta \left\{V + V^2 + 0{,}55V\frac{\rho^{/}}{\rho}\right\}, \tag{14}$$

$$V/_{t=0} = \frac{\varphi}{3U_0} - \frac{1}{3} \equiv \varphi(x, y, z).$$

Equation (14) is reduced, since it is written in dimensionless variables and holds for any "reduced model" coil. Substituting (14) into the known heat conduction equation [11]

$$c\rho U_t = \frac{\partial}{\partial Y}\left(\lambda \frac{\partial U}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\lambda \frac{\partial U}{\partial Y}\right) + \frac{\partial}{\partial Z}\left(\lambda \frac{\partial U}{\partial Z}\right).$$

The desired equation will be:

$$\left(1 + \frac{\rho^{/}}{\rho_0}\right)c\rho_0 V_t = 1{,}7 \cdot 10^{-2}\left\{\frac{\partial}{\partial Y}V_x(1+V) + \frac{\partial}{\partial Y}V_y(1+V)\right\}. \tag{15}$$

Because

$$t^{/} = t \cdot \frac{1{,}7 \cdot 10^{-2}}{\rho_0 \cdot c},$$

then

$$\left(1 + \frac{\rho^{/}}{\rho_0}\right)V_t^{/} = \left\{\frac{\partial}{\partial X}\left[(1+V)V_x^{/}\right] + \frac{\partial}{\partial Y}\left[(1+V)V_y^{/}\right] + \frac{\partial}{\partial Z}\left[(1+V)V_z^{/}\right]\right\}. \tag{16}$$

Equation (16) describes the propagation of heat in the coils. This equation is solved by the method of successive approximations.

Let the first approximation be

$$V_t^0 = \Delta\left(V^0\right),$$

$$V^0\big/_{t=o} = \varphi(x,y,z). \tag{17}$$

Equation (17) is solved by the known method [10].

$$V^0 = V^0(x,y,z) = \left(\frac{1}{2\sqrt{\pi t}}\right)^3 \int\int\int_{-\infty}^{+\infty} G\psi dx' dy' dz', \tag{18}$$

where

$$G = \exp\left(-\frac{\left(x-x'\right)^2 + \left(y-y'\right)^2 + \left(z-z'\right)^2}{4t}\right).$$

Solutions of equations (16) and (17) differ from each other by the addition of V^D,

$$V = V^0 + V^D,$$

where $V^D \ll V^0$. Substituting $V = V^0 + V^D$ into equation (16) and neglecting terms of the third order of smallness, let's obtain

$$V_t^D = \Delta V^D + \Delta\left\{\left(V^0\right)^2 - 0,45 V^0 \frac{\rho'}{\rho_0}\right\}, \tag{19}$$

$$V^D\big/_{t=o} = 0.$$

If let's assume that in the initial stage the heat distribution region is small, then the inhomogeneity of the raw cotton density can be ignored. Then equation (12) will have the form

$$V_t = \Delta\left[V\left(1+V\right)\right]. \tag{20}$$

If the heat distribution region is sufficiently large, then the value V is small and the linear equation has the following form.

$$\left(1+\frac{\rho'}{\rho_0}\right)V_t = \Delta. \tag{21}$$

It follows from equation (21) that the heat flux is more intensively propagated from denser layers of raw cotton to less dense ones. The value is found from the experimental data, for example, the dependence of the density of raw cotton on the coil height is determined by the formula

$$\rho = \rho_0(1-0,1z), \tag{22}$$

where z – the upward direction along the coil height.

In practice, the dependence of raw cotton density on the coordinates x, y is often encountered. Taking into account the slow variation of the density with respect to the coordinate, it can expand the functions ρ'/ρ_0 with respect to the coordinates x, y, z

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X, y, z in a Taylor series to terms of the third order of smallness:

$$\frac{\rho^{/}}{\rho_0} = A_0 x + B_0 y + C_0 z + A_1 x^2 + B_1 y^2 + C_1 z^2. \tag{23}$$

For $A_0 = 0$; $B_0 = 0$ the density distribution with respect to the central axis z is symmetric. From the equations (22) and (23)

$$\frac{\rho^{/}}{\rho_0} = -0,1z + A_1 x^2 + B_1 y^2, \tag{24}$$

where A_1, B_1 – coefficients that depend on the initial parameters of the coil. It can be seen from equations (21) and (24) that the raw cotton are heated more than in the surface layers in the central zones of the coil.

Similarly to equation (15), let's obtain the equation of heat propagation for the case of the bulk state of raw cotton. It differs from equation (16) in that

$$P_{av} = 700 \text{ N/m}^3$$

and ρ varies in the range 600÷1000 N/m³. So

$$\rho - \rho_0 \left(1 + \frac{\rho^{/}}{\rho_0} \right); \quad \frac{\rho^{/}}{\rho_0} < 0,25, \tag{25}$$

That is, the effect of the change in density on the propagation of heat in the bulk form is less than in the coils. In particular, the heat of propagation near the source can be considered to be small. Without loss of generality, let's consider the propagation of heat from the source in a homogeneous model at a temperature of 80 °C.

$$V_t = \frac{\partial}{\partial x} \left[(1 + V) V_x \right]. \tag{26}$$

It can be seen that the pocket spontaneous heating occurs locally, with sharp boundaries: the normal region is an overheated region.

Let's consider the propagation of heat away from the focus. Taking the center of the coil as the origin, let's expand the Taylor form of the function $\rho^{/} / \rho_0$ up to terms of the second order of smallness

$$\frac{\rho^{/}}{\rho_0} = A_0 x + B_0 y + C_0 z + A_1 x^2 + B_1 y^2 + C_1 z^2. \tag{27}$$

In the coils, the change in density relative to the central axis x and y is symmetrical. Taking into account the law of symmetry, let's obtain $A_0 = B_0 = 0$. From the experimental data:

$$C_0 = 0,1, \ C_1 = 0.$$

A_0 and B_0 are small compared to C_0, so A_0, $B_0 \ll C_0$

$$\left(1 + \frac{\rho^{/}}{\rho_0} \right) V_t = \Delta V. \tag{28}$$

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The equation is accessible equation of mathematical physics and is called the heat propagation equation in linear homogeneous media.

In practice, the initial temperature $\varphi(x, y, z)$ is discrete because the measurements of the temperature of raw cotton during storage are usually done pointwise.

Let a discrete initial temperature distribution is given. Then, using the superposition principle in the linear approximation, have for $V^0(x, y, z)$ the corresponding solution.

Let the coordinates of the source with the initial temperature $A_{m.n.k.}$ (**Fig. 2**) be in the volumes between the following coordinates:

$$x_1=m, \ x_2=m+1, \ m=0,1,2;$$

$$y_1=n, \ y_2=n+1, \ n=0,1,2;$$

$$z_1=k, \ z_2=k+1, \ k=0,1,2.$$

Let's introduce the notation $\vec{x}_{m.n.k} = (x_m + y_n + z_k)$.

Function $\psi(\vec{x})$ can be represented in the form as the unit function of Heaviside:

$$\Psi(\vec{x}) = \sum_{m.n.k} A_{m.n.k.} \cdot \theta_{m.n.k.} (\vec{x} - \vec{x}_{m.n.k.}) \cdot \theta_{m+1,n+1,k+1} \cdot (\vec{x}_{m+1,n+1,k+1} - \vec{x}) + A_0. \tag{29}$$

And $A_0=A_0=U_{min/}U_{cp}$, where A_0 is a constant.

Substituting equation (29) into expression (18) and using the properties of the Heaviside function, obtain

$$\Theta_{m,n,k}=\theta(x_m-x)\cdot\theta(y_n-y)\theta(z_k-z),$$

$$V^0=\left(\frac{1}{2\sqrt{\pi t}}\right)^3 \cdot \sum A_{m,n,k} \int_{x_m}^{x_{m+1}} \int_{y_n}^{y_{n+1}} \int_{z_k}^{z_{k+1}} G(\vec{x}\vec{x}^1) d\vec{x} + A_0. \tag{30}$$

As a result of the studies, the following conclusions can be drawn: the expression (16) is the general solution of the mathematical model of heat propagation in cotton raw during storage in coils, on the basis of which a number of physical real models can be constructed. For example, to diagnose thermal dynamics in coil using computer mathematical modeling of the heat propagation in raw cotton.

Thus, a mathematical model is proposed for investigating the heat propagation of a given initial temperature field. The model allows to preliminary estimate the likely picture. The temperature field in the given macro volumes of raw cotton, taking into account the nonlinear moisture and inhomogeneous properties of the medium (**Fig. 2**).
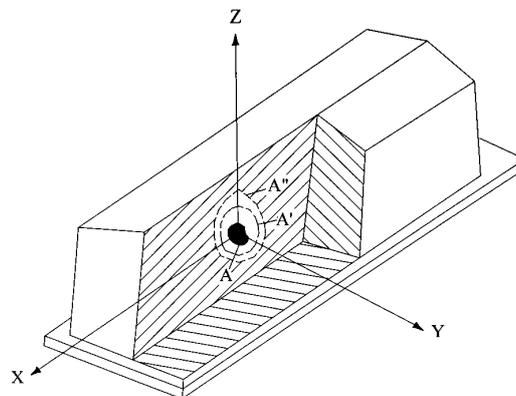

Fig. 2. The process of heat propagation from the source of spontaneous heating

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3. Discussion of research results

The paper presents the results of a study of the heat propagation of raw cotton during storage in coils.

The model allows to preliminarily give an estimation of spontaneous heating in raw cotton packed for storage in coils, thus creating conditions for the quality storage of raw cotton. On the other hand, it provides an opportunity to develop recommendations for conducting preventive measures.

The research is a continuation of the work carried out earlier and published in [12–14].

Further work to improve the method should be aimed at improving the methods of conducting experiments and software for processing the results of primary measurements.

5. Conclusions

1. As a result of experimental studies it is established that the heat flux propagates from dense layers of raw cotton to less dense layers at a constant rate.

2. A mathematical model of raw cotton is developed on the basis of experimental studies.

3. It is experimentally established that the thermal conductivity of dense layers of raw cotton is higher than that of raw cotton with a low bulk density.

4. Having determined the dependence of raw cotton on the coil height, the equation of which describes the propagation of heat in coil, it is also possible to use for the bulk state of cotton.

In the bulk form, the effect of density changes on the heat propagation is less than in the coils.

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