INTEGRATION OF KINEMATIC AND DYNAMIC MATHEMATICAL MODELS OF A TWO-AXLE ELECTRIC CAR IN THE PROBLEM OF ESTIMATING ITS STABILITY ON TURNS

***Vadym Horeniuk*[1]**

[1]Department of Electromechanical Systems Automation in Industry and Transport, Vinnytsia National Technical University, Vinnytsia, Ukraine
E-mail: gvv.ghost@gmail.com
ORCID: https://orcid.org/0000-0001-7494-3691

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ABSTRACT

Object of research: the process of movement of cars with internal combustion engines or the electric drive on a road curve.

Investigation problem: assessment of the stability of cars with internal combustion engines or electric drive on a road curve and determination of conditions of its ensuring.

The main scientific result. The article evaluated the stability of cars with internal combustion engines or electric drive on a road curve and determines the conditions of its ensuring using an algorithm that combines mathematical models of car movement on a road curve, synthesized based on balance equations of both kinematics and dynamics. The proposed models consider the change in speed of cars while driving on a road curve, and therefore belong to the class of differential equations. The analysis of these models allows calculating changes in time of values of limiting and critical speeds of movement of the car on a road curve. The article identifies the prospects of integration into this set of mathematical models another one, synthesized in the space of linguistic variables that characterize the uncertainty of the road surface and the degree of tire wear on different wheels of the car.

The area of practical application of the research results: Automotive enterprises specializing in equipping cars with traffic control systems.

Innovative technological product: A method of determining the limiting parameters of movement of the car on road curves, at which the car does not overturn while passing turns, and an algorithm for its implementation, which combines kinematic and dynamic mathematical models of car movement on the road curve.

Scope of application of the innovative technological product: Equipping cars with additional control systems that assess the critical values of the traffic parameters on turns to ensure the conditions of non-overturning when the car passes these turns.

1. Introduction

1. 1. The object of research

The object of research is the process of movement of cars with internal combustion engines or electric drive on a road curve and the definition of critical values of movement parameters at which the car keeps stability and does not skid and not overturns on this road curve.

1. 2. Problem description

Obvious, i.e., one that does not require additional justification, that during car movement along the horizontal section of a road, the most difficult is the stage of passing its turns. It is since without calculating the speed of entering a turn, the driver of an electric car can get off the road into a ditch. It can happen because of the action of the moment created by the centrifugal component of the speed as well as a car skidding because of the driver's incorrect assessment of the state of the road surface and the degree of wear of the wheel tires.

The study of the car overturning issue while driving on the turn is devoted to works [1, 2]. There, the authors determine the parameters of the car speed and roll angle at which can be avoided the car mass center shifting to the point at which overturns of the car will occur. The authors of [3, 4] solve the problem of preventing the car from skidding when cornering using nonlinear dynamic models of reactions of forces in the tires during sliding when cornering. In [5], in addition to models of tire force dynamics, the difference in speeds of non-driving wheels is also considered. However, all three works used a two-wheeler car model, which is simplified. Since the works [1–5] consider only or the car overturning or its skidding, so it was decided to develop a four-wheel car model that will consider both cases.

The work [6], using information taken from works [7] and [8], shows that the translational movement of the car is possible only if

$$F_R + F_S + F_C \le F_R^{kL-F} + F_R^{kL-R} + F_R^{kR-F} + F_R^{kR-R} + F_T^C, \tag{1}$$

$$M_C \le M_G, \tag{2}$$

where F_R, F_S, F_C, F_T^C – the forces (centrifugal, side, Coriolis, engine thrust component that balances Coriolis force), F_R^{kL-F}, F_R^{kL-R}, F_R^{kR-F}, F_R^{kR-R} – forces of the wheel rolling resistance (left front, left rear, right front, right rear), P – gravity force, M_C, M_G – torques created by centrifugal and gravity forces.

The work [6] also shows that in the case when inequality (2) holds, and equation (1) sign will change to the opposite, the car will start to skid. And when inequality (1) holds, and the equation (2) sign will change to the opposite, the car will overturn. At the same time, the minimum value of the linear velocity of the car as a function of the angular velocity ω concerning the center of the curve of the road curve at which skidding begins can be found from the equation obtained by replacing the inequality sign with the equality sign in the expression (1). And the minimum value of the linear velocity v^C of the car as a function of the angular velocity ω at which overturning begins can be found from the equation obtained by replacing the inequality sign with the equality sign in the expression (2). These functions with a similar sliding coefficient value $k_{**}(\omega, P)$ for each wheel are

$$v^L = -\omega(R+\Delta p) + \left(\frac{R+\Delta p}{m} \begin{pmatrix} k_{**}(\omega,P)P + F_T \operatorname{tg}\alpha - \\ -k_{FA}S_S\left(\omega L + v_a \sin(2\pi-\mu)\right)^2 - \\ -k_{SA}S_F\left(\omega L + v_a \sin(2\pi-\mu)\right) \end{pmatrix} \right)^{0.5} = f_1(\omega), \tag{3}$$

$$v^C = -\omega(R+\Delta p) + \left(\frac{R+\Delta p}{m} \begin{pmatrix} \frac{\Delta c}{h}P - k_{FA}S_S\left(\omega L + v_a \sin(2\pi-\mu)\right)^2 - \\ -k_{SA}S_F\left(\omega L + v_a \sin(2\pi-\mu)\right) \end{pmatrix} \right)^{0.5} = f_2(\omega), \tag{4}$$

where to the previously defined values are added: m the car mass, h – the height of car mass center, Δp – the distance between the car mass center and the inner arc of asphalt covering of a road curve, Δc – the distance between the car mass center and outer wheel when a car is moving along a road curve, R – the distance from a road curve center to the inner arc of the asphalt covering, v_a – the speed of the oncoming airflow of the car moving along a road curve, S_F, S_S – the car front and side areas, L – the car length, k_{FA}, k_{SA} – coefficients, and μ, α – the corners between v_a and v vectors with the tangent to the trajectory at the point with respect to which the balance equations are formed (1), (2).

Note: here are not provided the vector diagrams, characterized all forces and parameters of expressions (1)–(4) because they are shown in the works [6, 7], published in scientific journals that have their websites, and therefore it is possible to get acquainted with them at will, using the links included in references.

The analysis of the functions (3) and (4) showed they synthesized from the expressions (1), (2) by pre-converting these expressions into balance expressions, which are kinematic mathematical models of the car for the selected trajectory point at the chosen point of time.

As the authors showed in work [9], it is not enough to consider only the kinematics of a two-axle car to determine the driving conditions of its moving along the horizontal part of a road. Also, it is necessary to consider its dynamics and build a car movement mathematical model as the system of three second-order differential equations

$$\begin{cases} mx_0'' = F_{AX} + F_{BX} + F_{CX} + F_{DX}, \\ my_0'' = F_{AY} + F_{BY} + F_{CY} + F_{DY}, \\ J_0\varphi'' = M_A + M_B + M_C + M_D, \end{cases} \tag{5}$$

where m, J_0 – the car mass and inertia moment, normalized to the center of mass; φ – the car body longitudinal axis rotation's angle in the horizontal plane; x_0'', y_0'' – projections of car acceleration on the axes coordinate OX, OY; F_{AX}, F_{BX} F_{CX}, F_{DX}, F_{AY}, F_{BY} F_{CY}, F_{DY}, – the forces projections on the OX, OY axes, that acts on the wheels A, B, C, D; M_A, M_B, M_C, M_D – moments of forces that act on wheels relative to the car mass center.

1. 3. Proposed way to solve the problem

The described problem can be solved by developing a model of car motion by combining its kinematics and dynamics, which is shown in [9]. For this purpose, it is necessary to integrate into the mathematical models of the non-overturning conditions (3) and (4) on a road curve, obtained from balance equations (1) and (2) of its kinematics, the dynamic characteristics of a car by converting the mathematical model (5). And based on this model will be able to track changes in the values of the limiting and critical speed, at which, while driving along a road curve, the car will not be skidded or overturned when the car speed changes. Determining such values of the speeds on a road curve of the car is the purpose of this work.

2. Material and methods

Let's start from the fact that the balance equations (10) and (13) in [6], from which conditions (3) and (4) are obtained, which are based on the invariance of the speed at which the car, including electric traction, overcomes road curves. But most drivers on the road curves reduce the supply of fuel or electricity, which causes the linear speed of their car to decrease, accompanying this fall with the appearance of inertia, the vector of which is always directed in the direction opposite to these changes. Therefore, to the centrifugal force, which is trying to skid or overturn the car into a ditch, a projection onto the radius is also added – the vector of the point of the trajectory for which the balance equation is drawn up, the inertia force of this car, which is in a form similar to the left side of the second equation of the system (5), is added to the balance equation (10) in [6], because of which it takes the form:

$$m\frac{v^2+\omega^2(R+\Delta p)^2}{R+\Delta p}+\frac{1}{2}k_{FA}S_S\left(\omega L+v_a\sin(2\pi-\mu)\right)^2+k_{SA}S_F\left(\omega L+v_a\sin(2\pi-\mu)\right)+$$
$$+2mv\omega+m\frac{dv}{dt}\text{tg}\alpha=k_{**}(\omega,P)P+F_T\text{tg}\alpha, \tag{6}$$

and in a form similar to the left-hand side of the third equation of system (5) is added to the balance equation (13) in [6], because of which it takes the form

$$m\frac{v^2+\omega^2(R+\Delta p)^2}{R+\Delta p}+\frac{1}{2}k_{FA}S_S\left(\omega L+v_a\sin(2\pi-\mu)\right)^2+$$
$$+k_{SA}S_F\left(\omega L+v_a\sin(2\pi-\mu)\right)+2mv\omega+m\frac{dv}{dt}\text{tg}\alpha=\frac{\Delta c}{h}P, \tag{7}$$

Despite the equations (6) and (7), it is possible to see that, in contrast to the equations (10) and (13) from work [6], in which let's use them as generative, these equations already have the status of 1st order nonlinear differential equations with the following structure:

$$\frac{dv}{dt}=av^2+bv+c, \tag{8}$$

which differ only by coefficient "c" as coefficients "a" and "b" in them the same and have the following forms:

$$a = -\frac{1}{(R+\Delta p)\,\mathrm{tg}\alpha}, \tag{9}$$

$$b = -\frac{2\omega}{\mathrm{tg}\alpha}, \tag{10}$$

and coefficient "c" for the equation (6), let's mark them as c_6, which is given by the expression:

$$c_6 = \frac{1}{m\mathrm{tg}\alpha}\begin{pmatrix} k_{**}(\omega, P)P + F_T tg\alpha - m\omega^2(R+\Delta p) - \\ -\frac{1}{2}k_{FA}S_S(\omega L + v_a\sin(2\pi-\mu))^2 - k_{SA}S_F(\omega L + v_a\sin(2\pi-\mu)) \end{pmatrix}, \tag{11}$$

for the equation (7) – let's mark it as c_7, which is given by the expression:

$$c_7 = \frac{1}{m\mathrm{tg}\alpha}\begin{pmatrix} \frac{\Delta c}{h}P - m\omega^2(R+\Delta p) - \frac{1}{2}k_{FA}S_S(\omega L + v_a\sin(2\pi-\mu))^2 - \\ -k_{SA}S_F(\omega L + v_a\sin(2\pi-\mu)) \end{pmatrix}. \tag{12}$$

Let's transform equation (8) to the form

$$\frac{dv}{av^2+bv+c} = dt, \tag{13}$$

and, using the method described in [10], to the form:

$$\frac{1}{a}\frac{dv}{\left(v+\frac{b}{2a}\right)^2 + \left(\frac{c}{a}-\frac{b^2}{4a^2}\right)} = dt. \tag{14}$$

In the expression (14) let's perform replacing:

$$v + \frac{b}{2a} = \theta, \tag{15}$$

and get:

$$d\theta = dv. \tag{16}$$

Substituting expressions (15) and (16) into expression (14)

$$\frac{1}{a}\frac{d\theta}{\theta^2 + \left(\frac{c}{a}-\frac{b^2}{4a^2}\right)} = dt \tag{17}$$

or

$$\frac{1}{a}\frac{d\theta}{\theta^2 - \left(\frac{b^2}{4a^2}-\frac{c}{a}\right)} = dt. \tag{18}$$

Rely on expressions (9)–(12), let's determine which sign will have the expression $\left(\frac{b^2}{4a^2}-\frac{c}{a}\right)$.

Using expressions (9), (10), let's find, that

$$\frac{b^2}{4a^2} = \omega^2(R+\Delta p)^2 > 0, \tag{19}$$

and, using expressions (9), (11), (12), it is possible to see that

$$-\frac{c_6}{a} > 0, \tag{20}$$

$$-\frac{c_7}{a} > 0, \tag{21}$$

since the coefficient a has a negative sign, and coefficients c_6, c_7 when the speed approach to critical value in normal movement mode at which there may be skidding or overturning of the electric car, as can be seen from expressions (11), (12), have a positive sign.

Thus, accepting the next replacement

$$\left(\frac{b^2}{4a^2} - \frac{c}{a}\right) = k^2, \tag{22}$$

expression (18) can be rewritten in the next form

$$\frac{1}{a}\frac{d\theta}{k^2 - \theta^2} = -dt \tag{23}$$

and integrate using the table of integrals from [10].

As a result of integration, let's obtain the expression

$$\frac{1}{2ak}\ln\left|\frac{k+\theta}{k-\theta}\right| + C = -t, \tag{24}$$

substituting in which the values of the parameters:

$$\frac{1}{\sqrt{b^2 - 4ac}}\ln\left|\frac{\sqrt{b^2 - 4ac} + b + 2av}{\sqrt{b^2 - 4ac} - b + 2av}\right| + C = -t. \tag{25}$$

The constant of integration C in expression (25) is determined from the condition that at the moment the car leaves the beginning of the rounding of the road (t=0), its speed is equal to v_0, therefore, substituting this condition into expression (25), let's obtain

$$C = -\frac{1}{\sqrt{b^2 - 4ac}}\ln\left|\frac{\sqrt{b^2 - 4ac} + b + 2av_0}{\sqrt{b^2 - 4ac} - b + 2av_0}\right|. \tag{26}$$

And substituting expression (26) in expression (25), let's obtain the solution of differential equation (23) in the form

$$\frac{1}{\sqrt{b^2 - 4ac}}\left(\ln\left|\frac{\sqrt{b^2 - 4ac} + b + 2av}{\sqrt{b^2 - 4ac} - b + 2av}\right| - \ln\left|\frac{\sqrt{b^2 - 4ac} + b + 2av_0}{\sqrt{b^2 - 4ac} - b + 2av_0}\right|\right) = -t. \tag{27}$$

3. Results

Applying the corresponding transformations to the expression (27), as a result, let's obtain

$$v(t,c) = \frac{v_0\left(2a\sqrt{b^2 - 4ac}\left(1 + e^{-t\sqrt{b^2-4ac}}\right) + 2ab\left(1 - e^{-t\sqrt{b^2-4ac}}\right)\right) + 4ac\left(1 - e^{-t\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2 - 4ac}\left(1 + e^{-t\sqrt{b^2-4ac}}\right) - 2ab\left(1 - e^{-t\sqrt{b^2-4ac}}\right) - 4a^2 v_0\left(1 - e^{-t\sqrt{b^2-4ac}}\right)}. \tag{28}$$

Substituting in expression (28) instead of the parameter "c" its value c_6 from (11), let's find that

$$v^L = v(t, c_6) = v^L(t, \omega), \tag{29}$$

and substituting in expression (28) instead of the parameter "c" its value c_7 from (12), let's find that

$$v^C = v(t, c_7) = v^C(t, \omega). \tag{30}$$

4. Discussion of research results

The first thing to emphasize when analyzing the results is how to correctly use the solution of the differential equation (23) in the form (28), which for the non-skidding conditions of the car on the road curve takes the form (29) and for the non-overturning conditions of the car on the road curve takes the form (30). But first, let's recall that the algebraic balance kinematic equations for the forces acting on the car at the road curve took on the form of differential equations after admitting the possibility of changing the car speed at this curve. Thus, to the forces that acted on the car in a stationary mode at a constant speed, a dynamic component was added due to changes in the inertia force, which is proportional to the derivative of this speed. So, to reconcile both of these modes of car movement, it is necessary to determine the initial speed for expression (28) when used in the form (29) by expression (3), which is a solution to the balance equation obtained from expression (1), and when using expression (28) in the form (30), this initial speed must be determined by expression (4), which is a solution to the balance equation obtained from expression (2). And only after meeting these conditions, will be possible to track changes in the values of the limiting and critical speed, at which, while driving along a road curve, the car will not be skidded or overturned into a ditch when the car speed changes.

The second thing that needs to emphasize is that the results were obtained under the condition that the adhesion of all wheels to the road is the same. It is possible only when the road condition over its entire width is the same, as well as the degree of tire wear of all four wheels. If this does not correspond to reality, then, as shown in work [11], to the kinematic and dynamic models of car movement along the road curve, it is necessary to add a model in the form of an appropriate knowledge base in the space of linguistic variables characterizing the state of the road surface and the degree of tire wear of each of the wheels of the car.

And since the integration of this model with two, which are already integrated to analyze the car movement along a road curve, requires additional databases that still need to be formed, the process of integrating the linguistic model into a set of kinematic and dynamic models will be in the next work, which will be a continuation of this one.

5. Conclusions

By integrating mathematical models of kinematics and dynamics of the car, an algorithm has been developed to determine such values of the speed on a road curve of cars with both internal combustion engines and electric drive considering that not exceeding that speed makes it impossible to skid or overturn. These models consider the changes in the car speed when driving along a road curve and therefore belong to the class of differential equations. The analysis of these models allows calculating changes in time of values of limiting and critical speeds of car movement along a road curve. It also defines the parameters allowing integrating into this set of mathematical models other one determined by the conditions of impossible to skid or overturn while the car is driving along a road curve. The latter is synthesized in the space of linguistic variables that characterize the uncertainty of the road surface and the degree of tire wear on different wheels of the car.

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