THE BAYESIAN APPROACH TO ANALYSIS OF FINANCIAL OPERATIONAL RISK

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ABSTRACT

The article provides a short overview of methods for constructing mathematical models in the form of Bayesian Networks for modeling operational risks under conditions of uncertainty. Let's provide the sequence of actions necessary for creating a model in the form of the network, methods for computing a probabilistic output in BN, and give examples of using the tool to solve practical problems of operational financial risk estimation. The study results can be used by financial institutions as a tool for resolving specific practical issues of risk estimation.
The object of research: methods for constructing Bayesian Networks for modeling operational risk in financial institutions.
Investigated problem: modeling operational risk under conditions of uncertainty.
The main scientific results: overview of methods for constructing Bayesian Networks for modeling operational risk under conditions of uncertainty; the methodology in the form of sequence of actions necessary for creating the model in the form of the network; methods for computing a probabilistic output in BN; examples of applying such approaches to solve practical problems of operational financial risk estimation.
The area of practical use of the research results: The research results can be used in the following financial institutions: banking system, insurance and investment companies.
Innovative technological product: computer based decision support system, allowing for high quality modeling and estimation of operational risks.
Scope of the innovative technological product: the practice of usage the proposed models in financial organizations provides an evidence of their high efficiency in terms of formal description and estimation of operational risk.

1. Introduction

1. 1. The object of research

The object of research is represented by the operational risks, which is taking place in financial organizations such as banking system, investment companies etc.

The problem is touching upon correct formal description of situations leading to financial loss caused by operational risk. The description is required in the form of adequate mathematical models that allow to take into consideration possible uncertainties.

The proposed solution to the problem is based on the use of probabilistic mathematical models, namely Bayesian networks, which have such useful properties, that taking into consideration uncertainties and operating with expert knowledge.

A Bayesian network (BN) is a probabilistic model that uses a graphical representation (coding) of conditional probabilistic relationships (dependencies) between variables of the process under study, using a directed acyclic graph.

Today there are many examples of practical application of the models in the form of BN. In the paper [1] the problem is considered of knowledge modeling in adaptive testing of students from a given discipline. The structure of the course provides division of the domain into chapters, and each of the chapters, in turn, corresponds to a set of concepts. Testing includes a set of test tasks, each requiring proficiency in one or more concepts. In its turn, knowledge of each of the concepts may be necessary to perform one or more test tasks. This paper uses a Bayesian network with binary variables, mapped disciplines, topics, concepts, and questions (studies). The teacher sets conditional probabilities for variables.

It is noted in [2] that measuring the level of competence by means of their answers to test tasks is a typical task of probabilistic considerations. The two most frequent cases that cause uncertainty are referred to in the literature as slip and guess. Students may accidentally answer incorrectly to a question when they know the answers – it calls a slip. Students can also randomly guess the correct answer or write off a task. This case is called a guess.

The paper [3] showed an example of operational risk modeling, which is considered to determine the most likely sources of operating losses in their occurrence, using Bayesian programming technologies.

Knowledge about the subject area is defined as a set of random variables: the source of risk events (chance nodes), the event itself, and the consequences of the event implementation (utility nodes). Variables can take two states: "occurred=1" and "not occurred=0", each of which is considered as a random event.

The experts with deep knowledge of the analyzed business processes develop the topology of influence diagrams. The resulting model makes it possible to analyze the sensitivity of the expected value of operational losses to the sources of their occurrence.

In [4], on the basis of Bayesian network a model for the initial diagnosis of diseases manifested by high blood pressure has been developed. The model structure and parameters at the initial stage were determined based on literature data on the significance of complaints, symptoms, and other signs as indicators of the diagnosed disease. The indicators' significance for each diagnosis is considered during the evaluation of the network nodes' parameters, using the selected functions. The provided results of the model are consistent with the knowledge and practical experience of one of the authors. At the same time, a quantitative assessment of the posterior probability of possible diagnoses can serve as an additional tool for determining the final diagnosis or subsequent steps to clarify it. The quality of probabilistic estimates obtained for the model can be improved by adding different input nodes and training the network using statistical data.

In the study [5], a model is proposed for analyzing operational IT risks based on the Bayesian networks. This model allows predicting the amount of damage caused by IT risks, depending on the quality of the software, the qualifications of IT specialists, and the use of various testing methods. The model is accompanied by a practical example that solves the problem of direct Bayesian inference and performs sensitivity analysis, which allows getting a visual idea of the impact of individual variables on the amount of loss from IT incidents. The problem of calculating the inverse Bayesian inference for analyzing and determining the causes of risk events is solved.

The article considers some types of BN, model constructing procedure in the form of BN, and application examples to financial risk estimation.

1. 2. Problem description

The purpose of the study is in the following: to consider some types of Bayesian networks used to solve practical problems, and BN constructing procedure; to determine the possibility of hiring Bayesian approach to constructing mathematical models of financial risk, and show examples of building the models of Bayesian type for estimating operational financial risk.

1. 3. Suggested solution to the problem

Graph nodes of BN are random variables, and arcs are arbitrary dependencies of these variables, determined using conditional probability tables. The table of conditional probabilities of each node contains the probabilities of the states of this node under the condition of the previous states of the nodes [6]. **Fig. 1** shows examples of the generated graph:

BN models are used to model subject areas characterized by structural and statistical uncertainty. Uncertainties arise when it is impossible to select (evaluate) a model with an exact structure, or when there is insufficient information about the state of the process at the time of decision-making, when the mechanisms that determine the behavior of the system are random, or when the system operates under the combined influence of factors. Direct edges connect the vertices (nodes) of the BN, and each node is assigned to a specific probability function. Thus, a BN is a directed acyclic graph that does not contain a path that starts or ends at the same node [7].

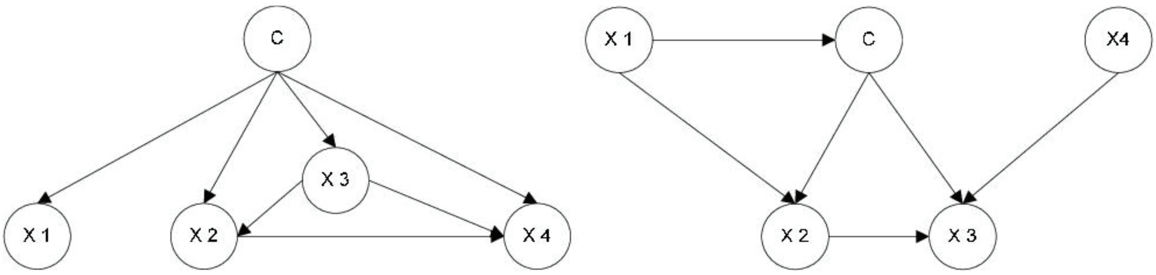


Fig. 1 Example of a Bayesian network structure

Regardless of the type of data used, there are statistical and dynamics BN, depending on the method of describing the modes of functioning of the objects of research. Each type of network is applied to a specific class of tasks [8].

Discrete networks are networks in which discrete quantities represent variables.

Continuous – networks in which variables are continuous quantities. Continuous BN models are used for stochastic modeling processes in the state space with continuous time [9].

Hybrid networks that have nodes with discrete and continuous variables.

Dynamic networks are networks where node values change over time. They are common in robotics and have demonstrated the potential for many data mining applications. For example, speech recognition, digital forensics, bioengineering, etc. They are used for modeling processes that change over time [10].

Dynamic Bayesian networks (DBNs) have been developed to unify and expand:

– traditional linear-state space models (such as Kalman filters);

– linear and normal forecasting models (for example, ARMA);

– and simple dependency models (like hidden Markov models),

in a general probable representation and inference mechanism for arbitrary nonlinear and non-normal time-dependent domains.

Dynamic databases are created to consider the dynamics of processes (their changes over time) and possible influences on their course. DBN is an extension of conventional (static) networks. First, let's construct an ordinary BN for the available variables, the structure of which is assumed to be invariant concerning time; that is why it remains constant. This structure is repeated for each subsequent time point with the arrival of new observations. In this way, the dynamics (changes in time) of the studied processes are reproduced.

2. Materials and Methods

Advantages of using DBN:

– DBNs use a tabular representation of conditional probabilities, making it easier to work with nonlinear processes.

– They generalize Markov models, which allows to represent the state space in a decomposed form instead of a single discrete random variable.

– Generalized Kalman filter algorithm, which allows arbitrary probabilities distribution, not only in the linear Gaussian problem statement.

– DBNs are used for adaptive filtering and support the decision-making of management decisions in automatic or semi-automatic modes of operation of control systems.

2. 1. Stages of building a model in the form of a BN

1. Object exploration (analysis of multidimensional processes): collecting information about the number and types of variables (discrete/continuous, parent/child); types of distributions; possible expert estimates; individual facts, etc.

2. Scaling and discretization of variables, creating a database.

3. Construction (evaluation) of structure(s): network topology generating with optimization algorithms (nodes, arcs).

4. Estimation of network parameters: determination of a priori probabilities, calculation of values of tables of unconditional and conditional probabilities for network variables, optimization of network topology.

5. Formulation (calculation) of probabilistic inference using alternative methods (algorithms).

6. Testing the network workability on well-known examples (data).

7. Apply the procedure for adopting the structure and parameters if necessary.

8. Practical application of the model, using the network for classification, presentation of results to the user.

Constructing the structure (topology) of a Bayesian network is shown in Fig. 2.

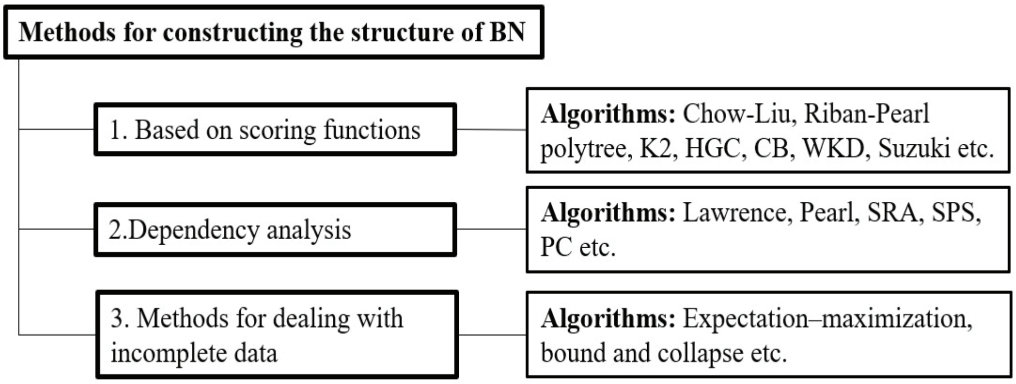
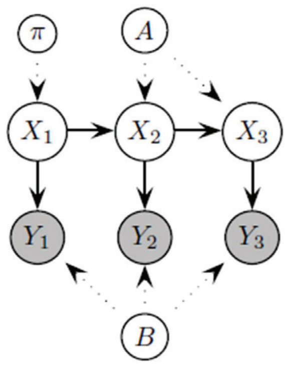


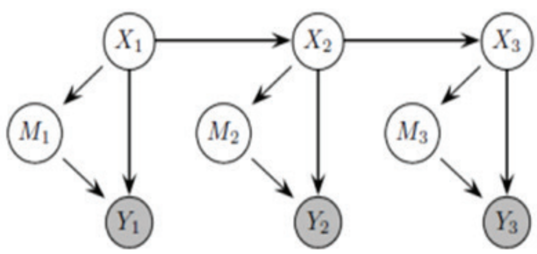
Fig. 2. Structure (topology) of a Bayesian network

2. 2. BN topology features

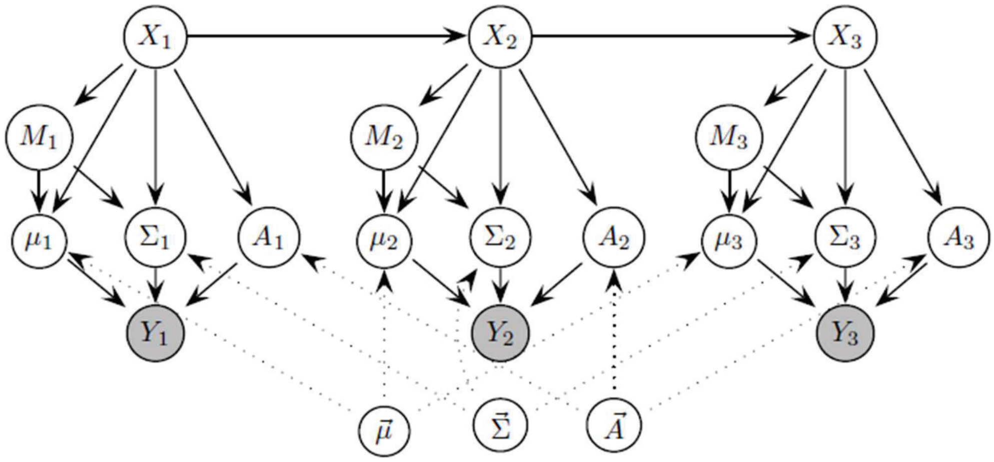
1. Hidden Markov network in the form of DBN



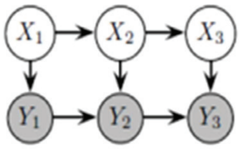
2. Hidden Markov models with mixed Gaussian output



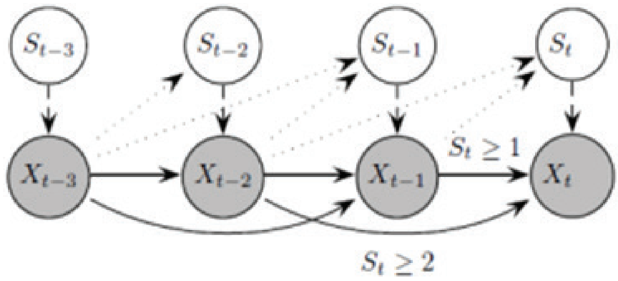
3. Hidden Markov models with semi-connected elements



4. Autoregressive hidden Markov models



5. Markov models with mixed memory



2. 3. Methods for generating the probabilistic inference

Among the methods for calculating probabilistic inference in BN, two groups of methods can be highlighted: approximating approach and calculating the exact conclusion [11]:

Algorithms for calculating the exact inference:

1. The Pearl algorithm.
2. Clique tree clustering algorithm.
3. Cutset conditioning algorithm (for determining the cross-section).
4. Variable elimination algorithms.
5. SPI – symbolic probabilistic inference.
6. Differentiated approach.

These algorithms give an accurate numerical result, but they are not used on large BNs when the network consists of hundreds or thousands of nodes due to the high computational complexity, close to exponential. Therefore, in cases of large BN, approximation algorithms are used for inference (losing the accuracy of calculations).

Approximation algorithms:

1. Algorithms for accurate determination of partial output.
2. Variational methods are used to calculate the average features of large networks.
3. Methods based on heuristic search algorithms that are used when moving from a probabilistic inference problem to an optimization problem.
4. Methods based on heuristic search algorithms that are used when moving from a probabilistic inference problem to an optimization problem.
5. Monte-Carlo methods.

3. Results

3. 1. Example 1

Considering Bayesian network to model fraudulent claims. Fraudulent claims may include cases of damage to property by the insured themselves or artificially inflated the amount of claim. The random variables used in the model are shown in **Table 1.**

Table 1

Bayesian Network vertices and their possible values

No.	Variable	Possible values
1	Underwriter experience	Senior, Junior
2	Branch reliance	Yes, No
3	Business volume	High, Low
4	Claims assessor experience	Senior, Junior
5	Random checks	Yes, No
6	Engage loss adjuster	Yes, No
7	Underwriting control	High, Low
8	Claims control	High, Low
9	Economic cycle	Up, Down
10	Fraudulent claim	Yes, No
11	Fraud detected	Yes, No
12	Cost of fraud	0, 50 000, 100 000, 150 000

Fig. 3 shows an example of the constructed model. All probabilistic distributions are resulted from expert evaluation.

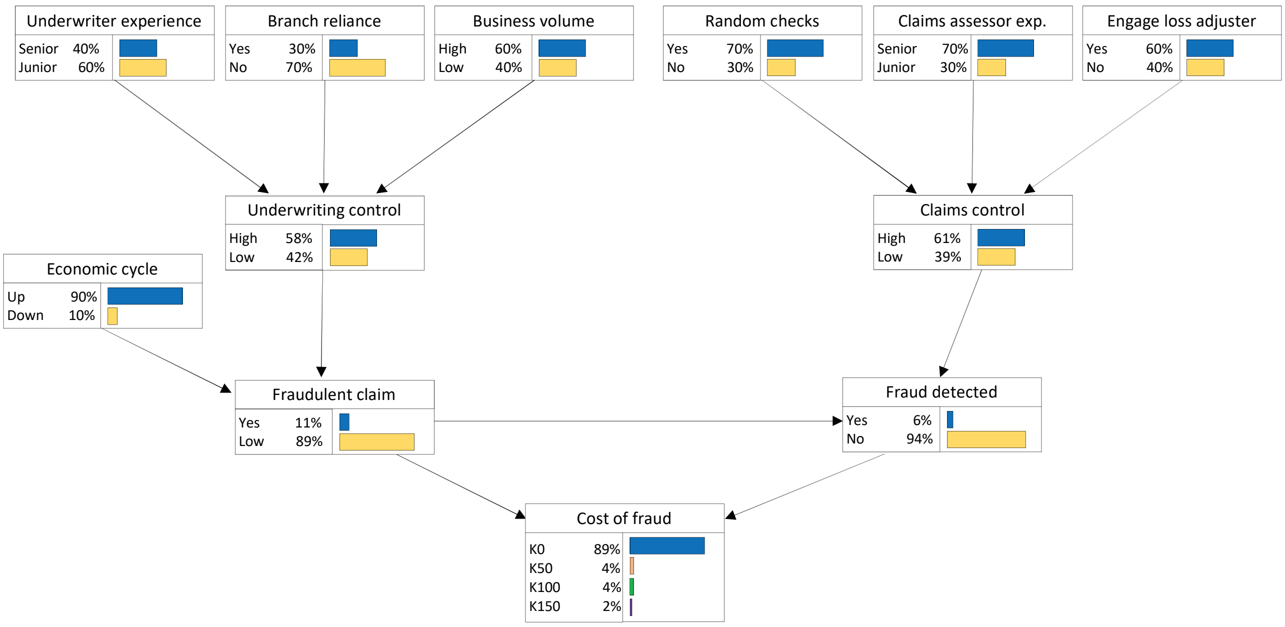


Fig. 3. Conditional probabilities of BN nodes

Using the model, the insurance company can decide on the amount of equity that should be allocated to protect the company from all cases of fraudulent claims.

In addition, the Bayesian network can be used to test various scenarios and help the manager optimize the risk profile. For example, if an insurance company wants to reduce costs by reducing the underwriter experience level to **Junior**, the effect of this action can be quickly investigated by changing the network parameter.

In this case, the insurance company can calculate the capital to cover losses caused by operational risks (CaR) at different confidence levels: 90 %, 95 %, 99 %.

Table 2
Capital Adequacy Requirements to cover OR with specified trust levels

Percentile, %	CaR
90	56,818
95	64,286
99	125,394

Table 3
Posterior values for scenario analysis

Variable	Value
Underwriter experience	Junior
Branch reliance	Yes
Business volume	High
Random checks	No
Claims assessor experience	Junior
Engage loss adjuster	No
Economic cycle	Down

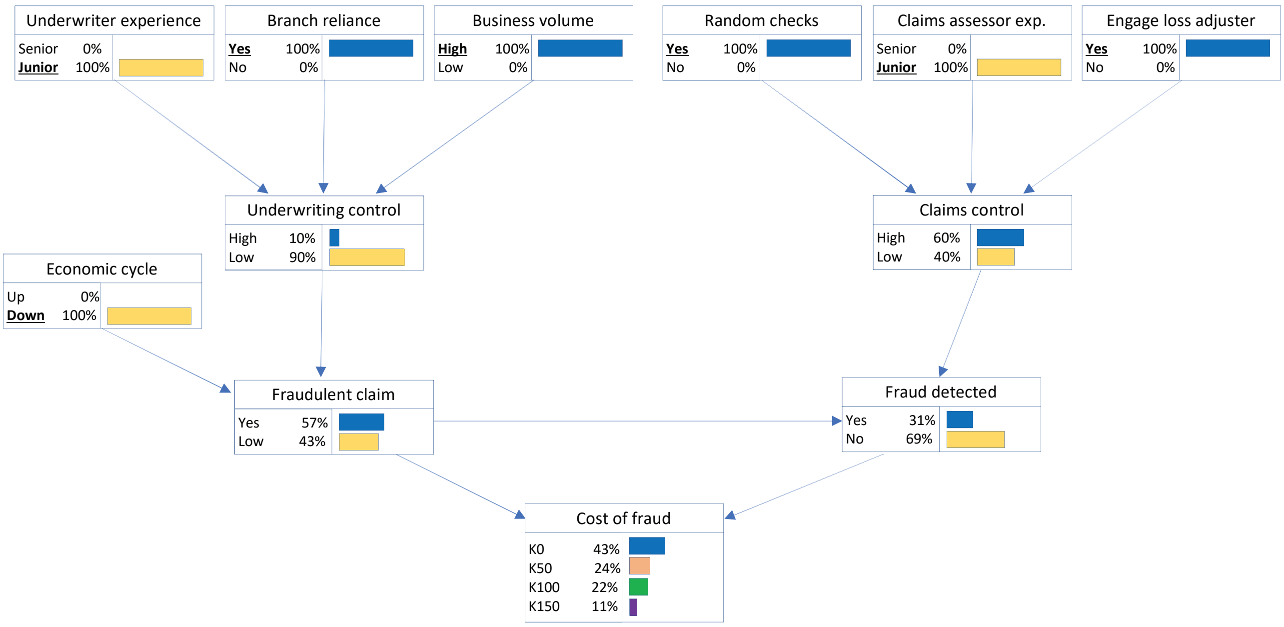


Fig. 4. Conditional probability distributions according to the scenario

Table 4
Capital to cover operational risk with specified confidence levels under the scenario

Percentile, %	CaR
90	94,374
95	119,243
99	143,849

It is possible to conclude that the compromise between lower costs increases the need for capital by 55 thousand units, assuming that the insurance company has a level of trust of 95 %.

3. 2. Example 2

Quality analysis of automatized client service (operational risk analysis): the case of discrete data and discrete parameters. Insurance company introduced automatized service for clients that provides for automatic registration of clients insuring their cars. The number of clients, users of the service, reaches several thousand per month. Consider the problem of estimating operational error θ after servicing *n* clients.

To simplify the problem statement suppose that θ can accept the following three values: 0.25 is good result; 0.50 is acceptable result; 0.75 is bad result that cannot be accepted. Available statistic shows that within former two years the company provided the automatized client service with the following quality:

– within 60 % of time the probability of service error was at the level of θ=0.25 (good result);

– within 30 % of time the probability of service error was at the level of θ=0.50 (acceptable result result);

– within 10 % of time the probability of service error was at the level of θ=0.75 (result that cannot be accepted).

These results were used as prior probabilities so that to forecast the level of service in the future. This distribution is given in the **Table 5**.

Table 5

Prior probabilities for parameter θ

Parameter and density for the errors	Service quality		
	good	acceptable	unacceptable
Probability of service errors (θ)	0.25	0.50	0.75
Density for the errors θ(p(θ))	0.60	0.30	0.10

After 10000 cases of client services the company decided to perform the service quality control. The control showed that out of ten randomly selected service cases two of them contained errors. What conclusion regarding the service quality should be made in this case? I.e., in other words, what is actual posterior distribution for the parameter, θ?

In this case the data has discrete form and first it is necessary to determine (say, on the bases of previous experience) type of distribution for data. On the basis of former experience It is possible to suppose that it has binomial distribution with parameter, θ:

$$f(\theta,\, n,\, r)=\binom{n}{r}\theta^{r}(1-\theta)^{n-r}=C_{n}^{r}\,\theta^{r}(1-\theta)^{n-r},$$

where $\binom{n}{r}=\dfrac{n!}{r!\,(n-r)!}$. The number of successful events is equal in this case to r=2; successful are events associated with emergence of errors in the client service out of 10 possible, i.e. n=10. Thus, likelihood function for data in this case is as follows:

$$L(\theta)=\binom{n}{r}\theta^{r}(1-\theta)^{n-r}=C_{10}^{2}\,\theta^{2}(1-\theta)^{10-2},$$

where θ=[0.25; 0.5; 0.75] is distribution of possible events linked to the number of service errors. The nominator of the Bayes rule in this case is as follows:

$$h(\theta\,|\,r,n)\propto L(r\,|\,\theta,n)\,g((\theta).$$

Now compute necessary likelihood values, L(θ):

– if θ=0.25, then $L(\theta)=\dfrac{n!}{r!(n-r)!}(0.25)^{2}(1-0.25)^{8}=0.30300;$

– if θ=0.50, then $L(\theta)=\dfrac{n!}{r!(n-r)!}(0.50)^{2}(1-0.50)^{8}=0.04405;$

– if θ=0.75, then $L(\theta)=\dfrac{n!}{r!(n-r)!}(0.75)^{2}(1-0.75)^{8}=0.00039.$

The prior and posterior densities for *h* are given in **Table 6.**

Table 6
Posterior probabilities

θ	Prior probabilities for θ	Likelihood $L(\theta)$	h=posterior density for θ
0.25	0.60	0.30300	0.955
0.50	0.30	0.04405	0.034
0.75	0.10	0.00039	0010
	Total sum:		0.999

Table 6 shows that the most probable value of θ, estimated on the basis of analysis of service quality in the case of automated servicing is the value: θ=0.25. It means that quality of service is on the "good" level (posterior probability for θ is 0.955). Before making final conclusion regarding quality of client service using automatized servicing system it is necessary to analyze larger number of cases than, 10; say 500 or 1000.

4. Discussion of research results

The provided study shows that probabilistic models can be successfully applied to formal description of the situations related to appearance of operational risk and its estimation. As disadvantage of the approach can be mentioned necessity of estimating prior probabilities that may require extra time and experiments for fulfilling the task.

The results of the study generally support previously performed studies in the sense that probabilistic Bayesian models can be applied successfully to modelling and estimation of operational financial risk, when the constructed model is adequate enough [3, 5]. The quality of the final result depends primarily on the quality of data and expert estimates.

The topic related to studying financial risks is rather broad and requires knowledge of mathematical modelling technologies in conditions of specific uncertainty. In turn, these specific uncertainties require separate studies to produce their formal descriptions that would allow to take them into consideration. The research limitations are touching upon these studies necessary to further improve the quality of final loss estimates.

The future studies are to be focused upon further improvement of modelling technology providing for mathematical instrumentation related to identification and taking into consideration possible set of statistical data uncertainties. The functions of the decision support system will be expanded with modelling nonlinear nonstationary processes related to financial risks.

5. Conclusions

Bayesian modeling and estimation procedures create a universal tool for solving most practical problems due to the ability to work with different data types that can be used in a particular case. They allow for analysis performing with incomplete data, as well as use them in cases where many factors influence the final choice. Examples considered in this study prove the possibility of applying Bayesian approach to modeling and estimation of financial risks. Depending on the quality of expert estimates available the approach based upon BN application helps to improve (decrease) possible operational loss for about 15–20 %. Also the time required for adjusting the model developed to new data is very short, not more than one hour what means that it has high adaptability feature in practice.

To provide high-quality results for application in many areas of human activity BNs building methodology need to be improved and adapted to the new problem, more accurate algorithms should be applied for constructing the network structure and calculating probabilistic inference.

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