PECULIARITIES OF MATHEMATICAL MODELING OF THE FLOW WHEN SELECTING THE GENERAL SIZES OF A BRIDGE CROSSING WITH GROUP HOLES

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Abstract: The problems of mathematical modeling of hydrodynamics of open flows are analyzed in the article when general sizes of bridge crossings with group holes are assigned. Most of the existing methods for calculating branched flows are based on the use of one-dimensional models, it does not allow to describe sufficiently the hydrodynamic processes occurring in zones with separated flows. Therefore, in the calculation of bridge crossings with group holes, two-dimensional models that take into account the features of the morphology of natural watercourses, the uneven distribution of the average velocities along the vertical, the phenomenon of flow separation, as well as the change in rate along the flow due to the separation of the flow is suggested. The existing experimental and theoretical studies are analyzed, the flow structure and the physical model of the flow branching process in the zone of influence of bridge crossings with group holes are considered. Based on the fundamental equations of flow transfer, a mathematical two-dimensional model of the flow of open flows is presented, which takes into account the influence of secondary currents. To substantiate the zone of influence of bridge crossings with group holes, an equation for vorticity is proposed that reflects the distribution of vortex structures. For the closure of the equations of motion and continuity, algebraic relations for the Reynolds stresses in conjunction with the two-parameter k-ε model for depth-averaged values are used, which is modified to take into account the effect of the curvature of the flow. The method of numerical realization of the proposed mathematical model is based on the predictor-corrector method using the McCormack scheme, modified by splitting the model equations with respect to spatial coordinates and time. The method of successive upper relaxation based on the Gauss-Seidel iterative method is used to realize the algebraic relations of turbulent stress transfer. The conclusion is made about the expediency of practical application of this model.

Keywords: bridge crossing, group holes, river flow, open flow branches, secondary currents, mathematical model, numerical modeling, turbulence, riverbed erosion, boundary conditions.

1. Introduction

It is difficult to overestimate the role of bridge crossings for transport in particular and the economy in general. However, flooding can cause deformations of the bottom in channel and floodplain areas, which can lead to the destruction of bridge supports, embankments of approaches, flowing dams and other hydraulic structures. Therefore, the correct assignment of bridge holes is an extremely important and urgent task.

The design of bridge crossings with group holes is particularly difficult. The phenomena of separation and fusion make significant changes in the hydrodynamic structure of the flow, which leads to the appearance of significant vortex areas, curvature of the flow in terms of development of secondary currents and changes in the level of the free surface (**Fig. 1**). A consequence of the influence of these phenomena is the abrupt change in the depth of the stream and, accordingly, the development of channel deformations that violate the normal operating conditions of bridge structures [1–3].

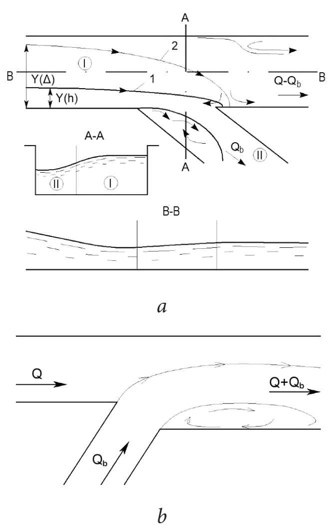


Fig. 1. Flow charts of open flow:
a – separation; *b* – junction

Carrying out full-scale experiments to study the properties of the flow during its flow through the holes of bridge crossings is very problematic. In addition, the obtained regularities allow to describe with a certain degree of accuracy this process only for a particular object.

The use of a mathematical model allows to determine the hydrodynamic field of velocities and pressures of a watercourse, which in turn will allow to correctly calculate the separation and erosion during flow separation. However, despite the considerable experience accumulated in the field of modeling open flows, the research is complicated by the fact that the general closed system of turbulent motion equations has not yet been formulated.

That is why the development of a mathematical model of the course of river flow on the sections of bridge crossings with group holes, taking into account the complex hydrodynamic structure of the open turbulent flow and the method of its implementation is an actual problem that is of scientific and practical interest.

2. Methods

The basis for the development of a mathematical model for the motion of a river flow in the zone of bridge crossing influence is the equation of the dynamics of a real liquid in “stresses”, that is, the Navier-Stokes equations. However, the existing analytical and numerical methods for solving these equations have been developed only for laminar liquid motion. And these are the simplest tasks that have limited practical application.

However, the flow in the zone of influence of the bridge crossing is the case of turbulent motion of a liquid, which is characterized by the fact that the velocities and pressures in the flow are not defined, but random functions of coordinates and time. Such parameters can’t be obtained from the Navier-Stokes equations in modern methods for solving these equations. Therefore, to solve the applied problems of hydrodynamics, approximate mathematical flow models are used, in which only the main factors are taken into account.

Using 3D models is a time-consuming and costly task. One-dimensional models do not take into account sufficiently all the factors that play an important role in the distribution of velocities, rates and pressures along the living section of the channel.

In this connection, the derivations of the equations of the two-dimensional hydraulics model from the equations of averaged three-dimensional turbulent motion in Cartesian coordinates are applied. The equations of the two-dimensional model are obtained by integrating the three-dimensional Reynolds equations vertically from the bottom mark to the free surface, that is, the depth of the flow:

$$\frac{\partial U_1}{\partial t}+\alpha_h\left(\frac{\partial U_1^2}{\partial x_1}+\frac{\partial U_1 U_2}{\partial x_2}\right)+\frac{\partial}{\partial x_2}\overline{V_1 u_2}=$$
$$=-g\frac{\partial H}{\partial x_1}-\frac{\partial}{\partial x_1}\overline{V_1^{'}V_1^{'}}-\frac{\partial}{\partial x_2}\overline{V_1^{'}V_2^{'}}-\frac{c_f U_1\sqrt{U_1^2+U_2^2}}{K_\phi h}, \qquad (1)$$

$$\frac{\partial U_2}{\partial t}+\alpha_h\left(\frac{\partial U_1 U_2}{\partial x_1}+\frac{\partial U_2^2}{\partial x_2}\right)+\frac{\partial}{\partial x_1}\overline{V_1 u_2}+\frac{\partial}{\partial x_2}u_2^2=$$
$$=-g\frac{\partial H}{\partial x_2}-\frac{\partial}{\partial x_1}\overline{V_1^{'}V_2^{'}}-\frac{\partial}{\partial x_2}\overline{V_2^{'}V_2^{'}}-\frac{c_f U_2\sqrt{U_1^2+U_2^2}}{K_\phi h}, \qquad (2)$$

$$\frac{\partial H}{\partial t}+\frac{\partial U_1 h}{\partial x_1}+\frac{\partial U_2 h}{\partial x_2}=0, \qquad (3)$$

where $\frac{\partial U_1}{\partial t}$ and $\frac{\partial U_2}{\partial t}$ – the inertial terms due to the local acceleration;

$$\frac{\partial}{\partial x_1}\overline{V_1^{'}V_1^{'}},\ \frac{\partial}{\partial x_2}\overline{V_1^{'}V_2^{'}},\ \frac{\partial}{\partial x_1}\overline{V_1^{'}V_2^{'}},\ \frac{\partial}{\partial x_2}\overline{V_2^{'}V_2^{'}}$$

are the terms of the turbulent stresses;

$$\frac{c_f U_1\sqrt{U_1^2+U_2^2}}{K_\phi h} \text{ and } \frac{c_f U_2\sqrt{U_1^2+U_2^2}}{K_\phi h}$$

are the terms that determine the tangential stresses at the bottom;

$$\frac{\partial}{\partial x_2}\overline{V_1}u_2,\ \frac{\partial}{\partial x_1}\overline{V_1}u_2,\ \frac{\partial}{\partial x_2}u_2^2$$

are terms that take into account the influence of secondary currents.

Secondary currents arising in the areas of flow curvature in terms of and heterogeneous channel roughness significantly affect the hydrodynamic structure of the flow [4–9]. This, in turn, affects the distribution of rates, the processes of channel formation and sediment transport. Secondary currents carry out transverse momentum transfer in the plan and with intensive transverse circulation, which occurs in the zone of influence of bridge crossings with group holes, this effect should be taken into account when creating a two-dimensional model.

According to the data of numerous studies [7, 11–13], it is expedient to use the k-ε model to close the two-dimensional model of turbulent flow motion, which describes the change in the turbulence averaged over the depth of the kinetic energy and the rate of its dissipation.

$$\frac{\partial k}{\partial t}+\overline{V}_j\frac{\partial k}{\partial x_j}=$$
$$=\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)+P-\varepsilon, \qquad (4)$$

$$\frac{\partial \varepsilon}{\partial t}+\overline{V}_j\frac{\partial \varepsilon}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right)+$$
$$+c_{\varepsilon_1}P\frac{\varepsilon}{k}-c_{\varepsilon_2}\frac{\varepsilon^2}{k}, \qquad (5)$$

where k and ε are, respectively, the kinetic energy and the rate of its dissipation.

With the aim of broader disclosure of the nature of turbulence and taking into account its anisotropic state, it is expedient to modify the k–ε model, it consists in its joint use with algebraic relations for Reynolds stresses. These expressions are obtained from the complete turbulent stress transfer equations by introducing model relationships and simplifying them.

To determine the boundaries of the turbulence distribution in the zones of separation and junction of river flows, it is proposed to use the vorticity equation.

Since we are considering the currents of open flows in the zones of their separation, it is necessary to take into account that the motion of the liquid in this case occurs with a variable flow along the flow, that is, has a non-consistent nature. For this purpose, an equation describing the change in depth in the compression zone of the flow is used in conjunction with the method for determining the plan of current lines. This allows to coordinate the decoupling on the separation or junction sections of the flows.

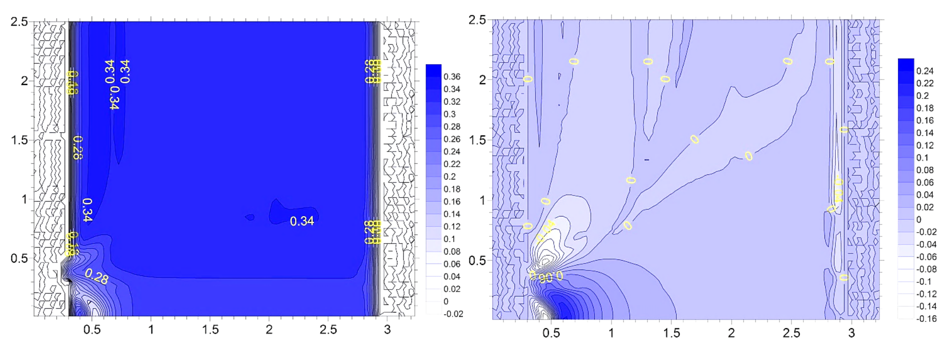
To realize the discrete analogs of the equations of internal flows and the k-ε model of turbulence, a finite-difference method of the predictor-corrector type is used, using the explicit McCormack scheme, modified by splitting the differential equations into one-dimensional spatial coordinates and time. This scheme is successfully used to solve problems of various classes [10]. Numerical realization of algebraic relations for turbulent stresses is carried out by the method of successive upper relaxation on the basis of the Gauss-Seidel method.

With the numerical implementation of this mathematical model, it is necessary to assign initial and boundary conditions for a realistic reproduction of the physical processes that occur in this case. These conditions significantly affect not only the stability of calculations, but also the accuracy of solving finite-difference equations.

3. Results

To verify and implement the proposed mathematical model, numerical calculations of the junction node of the experimental flows are performed. In this case, an experimental study of the junction node of open flows is used [8]. These calculations are carried out at the main channel Q_c=100 l/s, discharge rate Q_d=11.85 l/s and a flow depth h=18.57 cm.

The numerical calculations of the hydrodynamic structure of the experimental flow are compared with the results of the experiments. Analysis of the results shows sufficient convergence of the calculated and experimental data. The results of the calculations are shown in **Fig. 2.**



a *b*

Fig. 2. Graph of changes in hydrodynamic characteristics: *a* – graph of the change in the longitudinal velocity component U_x (m/s); *b* – graph o of the change in the transverse velocity component U_y (m/s)

Practical approbation of models and methods of their realization is carried out when calculating the kinematic structure of a separate flow in the zone of influence of a bridge crossing with group holes through the Styr river on the road Kyiv – Kovel near the Mayunichi village.

The results of the calculation of this bridge crossing are presented in the form of velocity diagrams (**Fig. 3**) and demonstrate the regularities of the current of the river flow. Comparison of the velocities obtained during hydrological surveys for the level of low-water waters with the calculated ones shows their satisfactory convergence.

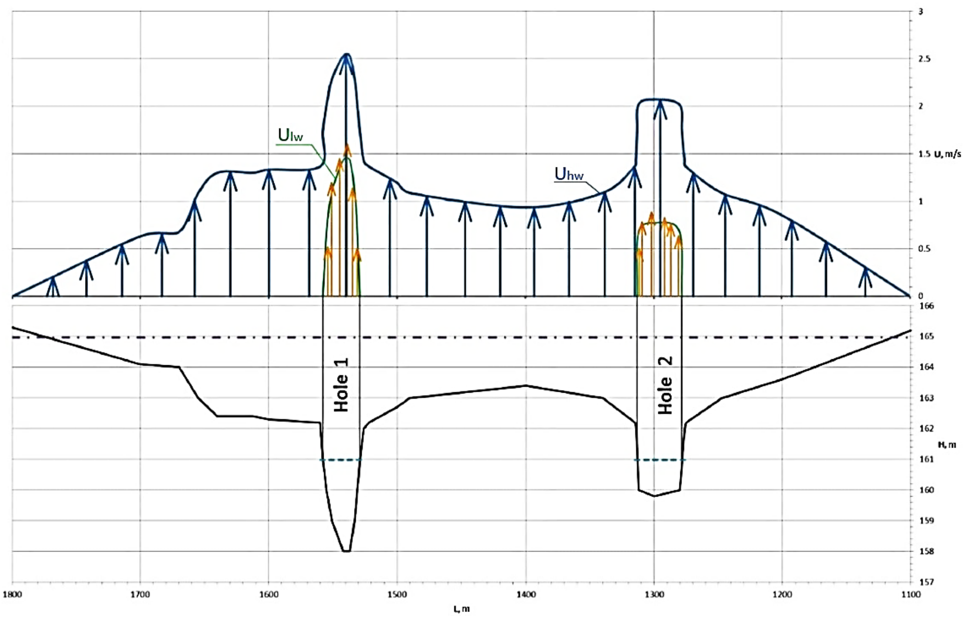


Fig. 3. Velocity diagrams in the bridge cross-section according to the results of calculations: U_{lw} and U_{hw} – respectively, the velocity diagrams of low and high waters; dotted line indicates the level of the water at the middle, dash-dotted line indicates the level of high waters, the arrows indicate the velocities

Thus, the proposed method of numerical implementation of this model allows to create an effective algorithm for solving the problem of assigning general dimensions of bridge crossings with group holes.

4. Discussion

The current of river flow in sections of bridge crossings with group holes has a complex hydrodynamic flow structure manifested in the curvature of the flow in the plan, the presence of significant circulation zones and the development of transverse circulation. During floods, the effect of these phenomena increases exponentially, which can lead to undesirable deformations of the bed, which disrupt the normal operation of hydraulic structures.

When calculating river flows, bridged by bridge crossings, the influence of secondary currents should be taken into account, since they play an important role in channel formation and sediment transport.

For a wide range of practical problems, it is advisable to use a two-dimensional model of river flow movement that takes into account the main factors affecting the formation of the field of velocities and flow pressures.

To close the two-dimensional equations of motion of a turbulent flow, it is advisable to use the k-ε model, which is quite popular due to its universality and relative simplicity. Algebraic relations for Reynolds stresses make it possible to simplify the model without solving the differential equations for these stresses. At the same time, they take into account the anisotropic state of the turbulent flow for its separation and junction in sections of bridge crossings with group holes.

Comparison of the calculated and experimental data indicates their satisfactory convergence and adequacy of the proposed model. The resulting mathematical model can be used to improve the technique for predicting erosion and the designation of general holes in bridge crossings.

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